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Richard B. Rood, Head
Data Assimilation Office
Goddard Space Flight Center
Greenbelt, Maryland

The GLA TOVS Rapid Algorithm Forward Radiance Modules and Jacobian Version 1.0

Meta Sienkiewicz

Data Assimilation Office, Goddard Laboratory for Atmospheres General Sciences Corporation, Laurel, Maryland

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Abstract

This document describes the rapid transmittance algorithm and forward radiative transfer calculations used in the Goddard Laboratory for Atmospheres (GLA) TOVS processing and the forward model and Jacobian program modules that have been derived from the TOVS processing routines for use by the Data Assimilation Office. An overview of the physical basis of the rapid algorithm is given, followed by descriptions of the routines for forward radiance calculation and routines for Jacobian calculation (derivative of the forward model with respect to input parameters). The source code for the modules is listed in the Appendix.

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List of Symbols	
ν frequency (cm ⁻¹ or GHz)	
$\phi_i(u)$ filter response function as a function of frequency for channel i	
$ au_{ u}(p, heta)$ transmittance, see (3)	
$ au_{iS}(heta, heta_S)$ atmospheric transmittance for solar radiation term	
heta zenith angle	
Θ_i brightness temperature, see , page 11	
$arepsilon_ u$ surface emissivity	
arphi solar zenith angle	
$B_{\nu}(T)$ Planck function, see (2)	
H_i solar radiation at the top of atmosphere	
$k_{\nu}(P)$ absorption per unit pressure	
R_{ν} radiance, see (1)	
R_i radiance for channel i , see (5)	
$R_i\downarrow$ effective downward flux of radiance from atmospheric emission	
T_g surface skin (ground) temperature	
T_{BB} cosmic background brightness temperature	
$R_i'(\theta)$ radiance reflected from surface, see (6)	

1 Introduction

One of the goals of the Data Assimilation Office (DAO) is to make better use of observations from new instruments, such as satellite-based sensors, which do not directly measure traditional atmospheric variables such as temperature and moisture. Formerly, height profiles (retrieved from satellite radiance measurements) were used in our data assimilation system in an ad hoc fashion without regard to the error covariance structure of the retrievals. New assimilation methodologies being evaluated by the DAO for using satellite measurements require knowledge of the forward radiative transfer model and its Jacobian (derivative with respect to input parameters). To this end, the code for the radiative transfer forward model for the TOVS (TIROS Operational Vertical Sounder) was extracted from the Goddard Laboratory for Atmospheres (GLA) TOVS retrieval program used by the Satellite Data Utilization Office¹ to process the TOVS Pathfinder Path A retrievals. This code was used to write a new set of Fortran modules for the forward model and corresponding Jacobian, which will be incorporated in future DAO assimilation systems.

The purpose of this office note is to document the program modules for the TOVS forward model and Jacobian. Section 2 describes the physics used in the forward model. In Section 3 the finite-difference representations used in the code are reviewed. These first sections, which give the scientific and mathematical background for the forward model and Jacobian, will be of particular interest to users who would wish to modify the code or to implement similar code using other forward model parameterizations. General users (those who wish to calculate TOVS brightness temperatures or Jacobians from input profiles of temperature, moisture and ozone) will find the description of the program interface in the latter sections useful: in section 4 the forward model program modules are discussed, and section 5 discusses the calculation of the Jacobian. Methods which use the forward radiative transfer model and Jacobian for assimilation of satellite measurements are discussed in Office Note 96–06 (Joiner and daSilva 1996).

2 Theoretical development

Physically-based retrievals and radiance assimilation methods use forward radiance calculations, *i.e.* calculation of radiance that would be measured in a satellite channel under given atmospheric conditions. The most accurate forward problem calculations are performed via *line-by-line* calculations to obtain high-resolution radiance spectra, which are convolved with the instrument response function to obtain channel radiances. (See section 2.2). Use of such computationally intensive calculations is not practical for operational retrievals or for data assimilation. Thus, rapid algorithms for forward calculation have been developed which use simple models, with coefficients fitted to results of line-by-line caluculation with representative atmospheric profiles.

The methodology for the forward calculation of radiance and brightness temperature using the GLA TOVS rapid algorithm is given in Susskind, et al. 1983, 1984 and references contained therein. A summary of the main points presented in those papers is given here. The purpose is to give users of the TOVS modules an idea of the physics underlying the code and to bring together information about the GLA forward model (as implemented in these modules) into one convenient location for easy reference.

¹now the GLA Sounder Research Team

2.1 Forward radiance calculation

The basic radiative transfer equation for radiation emitted at the top of the atmosphere at a single frequency ν can be written

$$R_{\nu}(\theta) = \varepsilon_{\nu}(\theta) B_{\nu}(T_g) \tau_{\nu}(p_s, \theta) + \int_{p_s}^{0} B_{\nu}[T(p)] \frac{\partial \tau_{\nu}(p, \theta)}{\partial p} dp + R_{\nu}'(\theta)$$
 (1)

where $\varepsilon_{\nu}(\theta)$ is the surface emissivity at zenith angle θ and frequency ν , T_g and T(p) are surface skin (ground) temperature and temperature at pressure level p respectively, p_s is surface pressure and $R_{\nu}'(\theta)$ is the contribution from radiance reflected from the surface (discussed in section 2.3).

 $B_{\nu}(T)$ is the Planck function or emitted radiance from a blackbody at temperature T for frequency ν under conditions of local thermodynamic equilibrium:

$$B_{\nu}(T) = \frac{2h\nu^3}{c^2(e^{\frac{h\nu}{kT}} - 1)} \tag{2}$$

where h is Planck's constant, c is the speed of light, and k is Boltzmann's constant. For microwave frequencies, blackbody radiation varies nearly linearly with temperature. Also, microwave measurements are expressed in terms of brightness temperatures, so B(T) is replaced by temperature T in the radiative transfer calculations.

The transmittance $\tau_{\nu}(p,\theta)$ from pressure p to the top of the atmosphere at zenith angle θ is given by

$$\tau_{\nu}(p,\theta) = \exp\left[-\int_{0}^{p} k_{\nu}(P)\sec(\theta)dP\right] \tag{3}$$

where $k_{\nu}(P)$ is the atmospheric absorption per unit pressure at frequency ν and pressure p.

2.2 Calculations for satellite channels

Each satellite channel represents an observation taken over a range of frequencies

$$R_i(\theta) = \int_{\nu} \phi_i(\nu) R_{\nu}(\theta) d\nu \tag{4}$$

where $\phi_i(\nu)$ is the filter response function for satellite channel i. For the GLA rapid algorithm Susskind et al. 1983 write this in a form similar to (1):

$$R_i(\theta) = \varepsilon_i(\theta) B_i(T_g) \tau_i(p_s, \theta) + \int_{r_s}^{0} B_i[T(p)] \frac{\partial \tau_i(p, \theta)}{\partial p} dp + R_i'(\theta)$$
 (5)

where now $B_i(T) = B_{\nu_i}(T)$, $\varepsilon_i(\theta) = \varepsilon_{\nu_i}(\theta)$ are evaluated at a prescribed central channel frequency ν_i and the transmittance τ_i is a modeled transmittance fitted to channel-averaged transmittances obtained from line-by-line calculations. Although the Planck function $B_{\nu_i}(T)$ changes with frequency ν it is assumed that the width of the channel response function ϕ is small enough that the Planck function at the central frequency ν_i can be used without modification.

2.3 Reflected radiance from surface

Susskind, et al. 1984 describe $R_i'(\theta)$, the contribution from reflected radiance, as:

$$R_i'(\theta) = (1 - \varepsilon_i) R_i \downarrow \tau_i(p_s, \theta) + \rho_i H_i \tau_{iS}(\theta, \theta_S)$$
(6)

where $R_i\downarrow$ is the effective downward flux of radiation from atmospheric emission for channel i, ρ_i is the bidirectional reflectance of the surface to the satellite of solar radiation from the sun with zenith angle θ_S , H_i is the solar radiation at the top of the atmosphere, and $\tau_{iS}(\theta,\theta_S)$ is the atmospheric transmittance along the entire path of incident and reflected solar radiation. The first term in the above equation represents the contribution of downwelling atmospheric radiation, and the second term represents the contribution of reflected solar radiation. The second term is calculated only for HIRS channels with frequencies greater than 2000 cm⁻¹ (HIRS 13 – 19), for solar zenith angles less than 85°.

For the first term, there are several ways that the effective downward flux is calculated, depending on whether the channel is an infrared (IR) or microwave (MW) channel. For HIRS IR channels 8,10,18,19 where the atmosphere is optically thin, the effective downward flux $R_i\downarrow$ is calculated as:

$$R_i \downarrow = 2\cos\theta \int_0^{\tau_i(p_s)} B_i(T) d\tau \tag{7}$$

As Susskind, et al. 1984 explain, this equation is based on the assumption of (a) an optically thin atmosphere, in which the downward flux at any zenith angle χ is proportional to $\sec(\chi)$, and (b) a Lambertian surface, for which the angular reflectance at any angle χ is a constant given by $(1-\varepsilon)/\pi$.

For the other HIRS infrared channels, the effective downward flux $R_i \downarrow$ is modeled according to Kornfield and Susskind (1977):

$$R_i \downarrow = F_i B_i(T_s) (1 - \tau_i(p_s, \theta)) \tag{8}$$

where T_s is the surface air temperature and F_i is a channel-dependent constant.

For the MSU microwave channels, the downward flux $R_i \downarrow$ is calculated explicitly:

$$R_i \downarrow = T_{BB} \tau_i(p_s, \theta) + \int_{p_s}^{0} T(p) \frac{\partial \tau_i(p, p_s, \theta)}{\partial p} dp \tag{9}$$

where T_{BB} is the background radiation from space from the big bang, and $\tau_i(p, p_s, \theta)$ is the (downward) transmittance between pressure level p and the surface. The downward transmittance can be related to the transmittance upward to the satellite as:

$$\tau_i(p, p_s, \theta) = \tau_i(p_s, \theta) / \tau_i(p, \theta)$$
(10)

which enables us to write (9) as:

$$R_i \downarrow = T_{BB} \tau_i(p_s, \theta) + \tau_i(p_s, \theta) \int_{p_s}^0 T(p) \frac{\partial}{\partial p} \frac{1}{\tau_i(p, \theta)} dp$$
 (11)

Strictly speaking, the relation in (10) is applicable only to monochromatic radiances; but as Weinreb *et al.* 1981 explain the microwave response functions are narrow and the transmittance across the channel varies slowly with frequency, so this formula may be sufficiently accurate for use.

2.4Rapid algorithm for transmittance

The rapid transmittance algorithm has the form

$$\tau_i(p_l, \theta) = \prod_{j=1}^l \tilde{\tau}_i(p_j, p_{j-1}, \theta)$$
 (12)

$$= \prod_{j=1}^{l} \tilde{\tau}_{iF}(p_j, p_{j-1}, \theta) \tilde{\tau}_{iO}(p_j, p_{j-1}, \theta) \tilde{\tau}_{iW}(p_j, p_{j-1}, \theta)$$
(13)

The total transmittance $\tau_i(p_l, \theta)$ between satellite level and a level l, is the product of effective layer transmittances $\tilde{\tau}_i(p_i, p_{i-1}, \theta)$ for all layers j between satellite level and the level l. These effective layer transmittances are modeled as the product of terms representing absorption due to atmospheric gases having a fixed mixing ratio $(\tilde{\tau}_{iF}(p_j, p_{j-1}, \theta))$, absorption due to ozone $(\tilde{\tau}_{iO}(p_j, p_{j-1}, \theta))$ and absorption due to water vapor $(\tilde{\tau}_{iW}(p_j, p_{j-1}, \theta))$. Coefficients for these terms are obtained by fitting transmittances obtained from line-byline calculations. Additional factors for the water vapor continuum absorption and aerosol scattering and absorption (not included in the line-by-line calculations) are also applied; these are presented at the end of this discussion.

As explained in Susskind et al. 1983, $\tilde{\tau}_{iO}(p_j, p_{j-1}, \theta)$ and $\tilde{\tau}_{iW}(p_j, p_{j-1}, \theta)$ are not single species layer transmittances calculated using ozone absorption or water vapor absorption alone. Since the channel radiance measurements are not monochromatic a simple product relation for transmittances does not apply. Given line-by-line transmittance calculations for $\tau_{iF}(p_l,\theta)$ (fixed gases only), $\tau_{iFO}(p_l,\theta)$ (fixed gases plus ozone), and $\tau_{iFOW}(p_l,\theta)$ (fixed gases, ozone, and water vapor), the effective layer transmittances are calculated according

$$\tilde{\tau}_i(p_j, p_{j-1}, \theta) \equiv \frac{\tau_i(p_j, \theta)}{\tau_i(p_{j-1}, \theta)}$$
(14)

$$\tilde{\tau}_{iO}(p_j, p_{j-1}, \theta) \equiv \frac{\tilde{\tau}_{iFO}(p_j, p_{j-1}, \theta)}{\tilde{\tau}_{iF}(p_j, p_{j-1}, \theta)}$$

$$(15)$$

$$\tilde{\tau}_{iO}(p_j, p_{j-1}, \theta) \equiv \frac{\tilde{\tau}_{iFO}(p_j, p_{j-1}, \theta)}{\tilde{\tau}_{iF}(p_j, p_{j-1}, \theta)}$$

$$\tilde{\tau}_{iW}(p_j, p_{j-1}, \theta) \equiv \frac{\tilde{\tau}_{iFOW}(p_j, p_{j-1}, \theta)}{\tilde{\tau}_{iFO}(p_j, p_{j-1}, \theta)}$$
(15)

The next two sections describe the models used for the effective mean layer transmittances in the GLA TOVS rapid algorithm. The values of the coefficients used with these models are derived by least-squares fitting to transmittances from line-by-line calculations performed using representative atmospheric profiles. The coefficients are provided for 71 layers in the vertical (see Table 1 for pressure levels bounding the layers) and at three zenith angles (0°, 50° , and 75°); values at other zenith angles are obtained by linear interpolation with respect to $sec(\theta)$:

$$\tilde{\tau}_i(p_j, p_{j-1}, \theta) = \tilde{\tau}_i(p_j, p_{j-1}, \theta_1) + \frac{\sec(\theta) - \sec(\theta_1)}{\sec(\theta_2) - \sec(\theta_1)} (\tilde{\tau}_i(p_j, p_{j-1}, \theta_2) - \tilde{\tau}_i(p_j, p_{j-1}, \theta_1))$$
(17)

where θ_1 and θ_2 are the rapid algorithm angles bracketing the zenith angle θ .

Table 1: GLA algorithm pressures, standard temperatures and scattering coefficients

Pressure	Temperature	Scattering	Pressure	Temperature	Scattering
(mb)	(K)	Coefficient	(mb)	(K)	Coefficient
0.10	221.6	0.000	240.0	222.9	0.00300
0.40	254.8	0.000	260.0	226.2	0.00300
0.70	264.4	0.000	280.0	229.3	0.00300
1.00	266.5	0.000	300.0	232.1	0.00300
1.30	265.0	0.000	320.0	235.2	0.00300
1.70	261.8	0.000	340.0	238.5	0.00300
2.00	259.7	0.000	360.0	241.6	0.00300
3.00	251.4	0.000	380.0	244.5	0.00300
4.00	246.1	0.000	400.0	247.3	0.00300
5.00	242.2	0.000	425.0	250.2	0.00300
6.00	239.2	0.000	450.0	253.3	0.00300
7.00	236.6	0.000	475.0	256.3	0.00300
8.00	234.4	0.000	500.0	259.0	0.00306
9.00	232.5	0.000	525.0	261.6	0.00312
10.0	230.8	0.000	550.0	263.9	0.00318
15.0	228.0	0.000	575.0	266.1	0.00325
20.0	224.5	0.000	600.0	268.2	0.00331
30.0	221.0	0.000	625.0	270.3	0.00340
40.0	217.0	0.000	650.0	272.2	0.00354
50.0	213.5	0.000	675.0	274.1	0.00368
60.0	210.6	0.00022	700.0	275.9	0.00381
70.0	208.1	0.00058	725.0	277.5	0.00409
80.0	206.8	0.00091	750.0	278.9	0.00438
90.0	206.5	0.00121	775.0	280.3	0.00446
100.0	206.1	0.00143	800.0	281.6	0.00501
110.0	206.6	0.00167	825.0	282.9	0.00560
120.0	207.9	0.00186	850.0	284.2	0.00619
130.0	209.0	0.00207	875.0	285.3	0.00678
140.0	210.1	0.00222	900.0	286.3	0.00741
150.0	211.0	0.00238	925.0	287.2	0.00864
160.0	212.2	0.00254	950.0	288.2	0.00988
170.0	213.4	0.00271	975.0	289.1	0.01112
180.0	214.7	0.00292	1000.0	290.0	0.01236
190.0	215.8	0.00300	1025.0	290.8	0.01359
200.0	216.9	0.00300	1050.0	291.7	0.01483
220.0	219.3	0.00300			

2.4.1 Ozone and water vapor contributions

The effective layer transmittances due to ozone and those due to water vapor are both modelled using an equation of this form:

$$\tilde{\tau}_{ic}(p_j, p_{j-1}, \theta) = \exp\left(-A_{i,j,c}(\theta)[1 - B_{i,c}(\bar{T}_j - 273)]u_c(j, j-1)^{N_{i,c}}\right)$$
(18)

where c is the constituent (either ozone or water vapor), $u_c(j,j-1)$ is the integrated column density of the species in the layer between levels j and j-1, $N_{i,c}$ is a channel and species dependent constant between 0.5 and 1, $A_{i,j,c}$ is an effective channel, species, pressure, and angle dependent absorption coefficient, $B_{i,c}$ is a channel and species dependent constant representing percent change per degree, and \overline{T}_j is the mean temperature in the layer between p_{j-1} and p_j .

Susskind et al. 1983 give the rationale for this formulation as follows: the effective layer transmittances $\tilde{\tau}_{iO}$ and $\tilde{\tau}_{iW}$ can be treated as having the transmittance properties of a gas in a homogeneous layer with mean temperature T, and pressure p of the atmospheric layer, and vertical column density u of the absorbing gas in the layer. The use of (14) removes most of the dependence of the mean layer transmittance on the properties of the atmosphere above the layer. Thus we would expect that the log of the mean layer transmittances would be proportional to u for weakly absorbing lines and $u^{1/2}$ for strong lines, with an exponent of intermediate value applicable to a composite of lines.

2.4.2 Fixed gas contribution

The definition of effective mean layer transmittances in (14) reduces the dependence of the layer transmittances on the profile above the layer, but this effect still must be taken into account in the model for fixed gas contribution. The transmittance in layer j for two profiles having the same layer temperature T_j but different temperature profiles above p_j will still differ because the profile with more attenuation above the level will have less effective attenuation within the level. Susskind et al. 1983 note that temperatures in regions of the atmosphere which do not contribute to the channel radiance are irrelevant – either no attenuation is taking place in those levels or the total transmittance is already small enough that second-order effects are not important. Thus, they define an effective mean temperature above pressure p_j as the average temperature weighted by the weighting function for each channel i:

$$\tilde{T}_{ij}(\theta) = \frac{1}{1 - \tau_i^{\circ}(p_j, \theta)} \int_0^{p_j} T(p) \frac{\partial \tau_i^{\circ}(p_j, \theta)}{\partial p} dp$$
(19)

where $\tau_i^{\circ}(p_j, \theta)$ is the transmittance of channel i for the standard temperature profile.

The effective mean layer transmittance is modeled according to

$$\tilde{\tau}_{iF}(p_j, p_{j-1}, \theta) = A_{ij}(\theta) + B_{ij}(\theta)(\bar{T}_j - \bar{T}_j^{\circ}) + C_{ij}(\theta)(\tilde{T}_{ij}(\theta) - \tilde{T}_{ij}^{\circ}(\theta))$$
(20)

where \bar{T}_j is the mean temperature in layer j between p_j and p_{j-1} for the given profile, \bar{T}_j° is the mean layer temperature for a standard temperature profile for the layer j and \tilde{T}_{ij} and \tilde{T}_{ij}° are the effective mean layer temperatures as defined in (19) for the given temperature profile and for the standard temperature profile, respectively.

2.4.3 Water vapor continuum

We now discuss the transmittance terms that are not included in the line-by-line calculations; the water vapor continuum and aerosol scattering terms. The water vapor continuum term is modeled as described in Susskind and Searl (1978), based on a representation given by Bignell (1970). The net atmospheric absorption in layer j due to the e-type component of the water vapor continuum is written

$$K_{H_2O}(\nu, j, j-1) = k(\nu, \bar{T}_j)\bar{e}_j\hat{u}_{H_2O}(j, j-1)$$
(21)

where $k(\nu, \bar{T}_j)$ is the absorption coefficient (g⁻¹ cm² atm⁻¹) for the e-type H₂O continuum component for layer j, \bar{e}_j is the mean H₂O partial pressure for layer j, and $\hat{u}_{H_2O}(j, j-1)$ is the H₂O column density (g cm⁻¹). The coefficient $k(\nu, \bar{T}_j)$ is assumed to vary linearly with temperature.

2.4.4 Scattering and absorption by aerosols

The term for aerosol scattering and absorption used with the GLA rapid transmittance algorithm is written as (Susskind *et al.* 1983):

$$\tau_{a_i}(p,\theta) = \exp(-k_{a_i}(p)\sec\theta) \tag{22}$$

where $k_{a_i}(p)$ is the sum of the aerosol scattering and aerosol absorption optical thickness from pressure p to the top of the atmosphere for the given channel i. For the 15- μ m channels, the total optical depth of the tropospheric aerosols is taken as 0.01 for nadir viewing, falling off with a 1.2-km scale height. There is an additional stratospheric aerosol layer between 12 and 20 km with a total optical depth 30% of that of the tropospheric layer. For the 4.3- μ m channels, the optical depths are taken to be a factor of 3 greater than the 15- μ m channels (i.e. a total nadir optical depth of 0.039). The values of scattering coefficients used are listed in Table 1.

3 Finite-difference representations

The purpose of this section is to relate the physical background of the forward model (as discussed in Section 2 to the finite difference representations that are used in the code for the forward radiative transfer calculation. From the finite-difference forward model representation we can perform derivative operations on the forward model to obtain the Jacobians for temperature, moisture, and ozone.

3.1 Finite-difference forward calculation

The description of the finite-difference forward model calculation in this section will follow the same steps as in the forward module code. Effective layer transmittances are calculated at the rapid algorithm zenith angles and interpolated to the satellite zenith angle. The product of the layer transmittances is taken to obtain the transmittance between the satellite and the rapid algorithm levels. The radiance corresponding to the input profiles is then calculated by taking vertical integral of the Planck function weighted by the derivative of the transmittance.

The notation convention used in this section is relatively simple. The subscripts of terms are ordered as (channel, level or layer, angle) if applicable. The channel index is usually i, and the angle index is k. Level indices used include j, l, and m. Thus, T_m is temperature at pressure p_m , \bar{T}_m is mean layer temperature for the layer m between p_{m-1} and p_m , $\tilde{\tau}_{i,j,k}$ is the effective layer transmittance for channel i, layer j, and rapid algorithm zenith angle θ_k , and $\tau_{i,l}(\theta)$ is transmittance for channel i between the satellite and pressure level l for zenith angle θ .

Fixed gas contribution In the forward model code, we first calculate the contributions to effective layer transmittance due to fixed gases, water vapor, and ozone. We consider the fixed gas contribution first. The expression for effective mean temperature defined in (19) can be written in finite difference form as:

$$\tilde{T}_{i,j,k} = \frac{1}{1 - \tau_{i,j,k}^{\circ}} \sum_{m=1}^{j} \bar{T}_m \left(\tau_{i,m-1,k}^{\circ} - \tau_{i,m,k}^{\circ} \right). \tag{23}$$

where $\tilde{T}_{i,j,k}$ is the effective mean temperature for channel i at pressure level j for rapid algorithm zenith angle k. The transmittance for the standard temperature profile $\tau_{i,j,k}^{\circ}$ is derived from (13) and (20):

$$\tau_{i,j,k}^{\circ} = \prod_{l=1}^{j} \tilde{\tau}_{F_{i,l,k}} = \prod_{l=1}^{j} A_{i,l,k}$$
 (24)

This can be substituted into (23) to get

$$\tilde{T}_{i,j,k} = \frac{1}{1 - \prod_{l=1}^{j} A_{i,l,k}} \sum_{m=1}^{j} \bar{T}_{m} (1 - A_{i,m,k}) \prod_{l=1}^{m-1} A_{i,l,k}
= \frac{1}{1 - \prod_{l=1}^{j} A_{i,l,k}} \sum_{m=1}^{j} W_{i,m,k} \bar{T}_{m}$$
(25)

where $W_{i,m,k} = (1 - A_{i,m,k}) \prod_{l=1}^{m-1} A_{i,l,k}$ is a coefficient which combines the constant terms of the vertical integration. Then (20) can be rewritten as

$$\tilde{\tau}_{F_{i,j,k}} = A_{i,j,k} + B_{i,j,k} (\bar{T}_j - \bar{T}_j^{\circ}) + \frac{C_{i,j,k}}{1 - \prod_{l=1}^{j} A_{i,l,k}} \sum_{m=1}^{j} W_{i,m,k} (\bar{T}_m - \bar{T}_m^{\circ})
= A_{i,j,k} + B_{i,j,k} (\bar{T}_j - \bar{T}_j^{\circ}) + C_{i,j,k}^* \sum_{m=1}^{j} W_{i,m,k} (\bar{T}_m - \bar{T}_m^{\circ})$$
(26)

where we now define a new coefficient $C^*_{i,j,k} = \frac{C_{i,j,k}}{1-\prod_{j=1}^{j}A_{i,l,k}}.$

Water vapor contribution The water vapor contribution to transmittance is made up of two terms, the modeled water vapor transmittance from (18) fitted from line-by-line transmittance calculations, and the water vapor continuum term given in (21). Since the coefficient $k(\nu, \bar{T}_j)$ is assumed to be a linear function of temperature we write the absorption for channel i in layer (p_{j-1}, p_j) as

$$K_{H_2O_{i,j}} = k_i^{\circ} \left(1 + \frac{dk}{dT} (\bar{T}_j - T^c) \right) \bar{e}_j \hat{u}_{H_2O_j}$$
 (27)

The contribution to transmittance from the water vapor continuum term would then be

$$\tilde{\tau}_{\text{cont}_{i,j,k}} = \exp(-\sec(\theta_k) K_{H_2O_{i,j}}) \tag{28}$$

The input water vapor profile is in terms of specific humidity (kg H_2O/kg air) so conversions to mean layer vapor pressure \bar{e} (atm) and column density \hat{u} (g cm⁻²) must be performed. For column density, the conversion is:

$$\hat{u}_{H_2O_j} = \rho_{H_2O_j} \Delta z_j = \rho_{\text{air}_j} q_j \Delta z_j = \frac{10 \Delta p_j}{g} q_j = \frac{10(p_{j-1} - p_j)}{g} q_j$$
 (29)

For H₂O vapor pressure the conversion is:

$$\bar{e}_{j} = \frac{\text{air mol. wt}}{\text{H}_{2}\text{O mol. wt.}} \frac{\bar{p}_{j}(\text{mb})}{1013.25 \frac{\text{mb}}{\text{atm}}} q_{j}$$

$$= \frac{1}{\epsilon} \frac{\bar{p}_{j}(\text{mb})}{1013.25 \frac{\text{mb}}{\text{oth}}} q_{j} \qquad (30)$$

For the transmittance modeled from line-by-line calculations (18), the coefficients are adjusted on input to be used with water vapor column density. We may write

$$\tilde{\tau}_{W_{i,j,k}} = \exp\left(-D_{i,j,k}[1 - E_{i,k}(\bar{T}_j - 273)]\{(\hat{u}_{H_2O_j})^{M_i}\}\right) \tag{31}$$

and this may be combined with the water vapor continuum term to obtain a term for the total water vapor contribution to effective layer transmittance

$$\tilde{\tau}_{H_2O_{i,j,k}} = \tilde{\tau}_{W_{i,j,k}} \tilde{\tau}_{\text{cont}_{i,j,k}}
= \exp\left(-D_{i,j,k} [1 - E_{i,k}(\bar{T}_j - 273)] \{(\hat{u}_{H_2O_j})^{M_i}\} - \sec(\theta_k) K_{H_2O_{i,j}}\right)$$
(32)

Ozone contribution For the ozone contribution the only factor is from the modeled transmittance (18). Since the coefficients for ozone are adjusted to be used with the ozone column density input to the routine, no conversions are necessary. Thus, we may write

$$\tilde{\tau}_{O_{3i,j,k}} = \exp\left(-F_{i,j,k}[1 - G_{i,k}(\bar{T}_j - 273)]\{(\hat{u}_{O_{3j}})^{N_i}\}\right)$$
(33)

Total transmittance When the contributions to effective layer transmittance from the fixed gases, water vapor, and ozone have been calculated, the effective layer transmittance is formed from their product:

$$\tilde{\tau}_{i,j,k} = \tilde{\tau}_{F_{i,j,k}} \tilde{\tau}_{H_2O_{i,j,k}} \tilde{\tau}_{O_{3i,j,k}} \tag{34}$$

The effective layer transmittances for the rapid algorithm angles that bracket the satellite zenith angle are interpolated linearly with respect to $\sec(\theta)$ as shown in (17). The product of the effective layer transmittances gives the total transmittance between the satellite and the rapid algorithm levels. For infrared channels, this is also multiplied by a factor $\tau_{a_{i,j}}$ (from (22)) to account for scattering. Thus, the total transmittance is:

$$\tau_{i,j}(\theta) = \tau_{a_{i,j}}(\theta) \prod_{l=1}^{j} \tilde{\tau}_{i,l}(\theta)
= \tau_{a_{i,j}}(\theta) \prod_{l=1}^{j} \left\{ \tilde{\tau}_{i,l,k} + \frac{\sec(\theta) - \sec(\theta_k)}{\sec(\theta_{k+1}) - \sec(\theta_k)} (\tilde{\tau}_{i,l,k+1} - \tilde{\tau}_{i,l,k}) \right\}$$
(35)

where θ_k and θ_{k+1} are the rapid algorithm angles bracketing the desired angle θ .

The transmittance for solar radiation reflected from the surface is calculated in the same way as the total transmittance in (35) except that transmittance is interpolated to the effective solar zenith angle $\theta_{EFF} = \sec^{-1}(\sec(\varphi) + \sec(\theta))$ (where φ is the solar zenith angle). Thus

$$\tau_{iS}(\theta_{EFF}) = \tau_{a_{i,N}}(\theta_{EFF}) \prod_{l=1}^{N} \left\{ \tilde{\tau}_{i,l,k} + \frac{\sec(\theta_{EFF}) - \sec(\theta_k)}{\sec(\theta_{k+1}) - \sec(\theta_k)} (\tilde{\tau}_{i,l,k+1} - \tilde{\tau}_{i,l,k}) \right\}$$
(36)

Forward radiance calculation The forward radiance calculation from (5) can be written in finite difference form as

$$R_i(\theta) = \varepsilon_i(\theta) B_i(T_g) \tau_{i,N}(\theta) + \sum_{l=1}^N B_i(\bar{T}_l) \left(\tau_{i,l-1}(\theta) - \tau_{i,l}(\theta) \right) + R_i'(\theta)$$
(37)

where ε_i is the surface emissivity for channel i and T_g is the surface skin temperature. Note again that for microwave channels we use \bar{T}_l in place of $B_i(\bar{T}_l)$

The expressions for reflected radiance $R_i'(\theta)$ are discussed in section 2.3. The basic equation (6) expressed this as the sum of terms for contribution from the downward flux of radiation from the atmosphere and a solar term (where appropriate).

$$R_i'(\theta) = (1 - \varepsilon_i) R_i \downarrow \tau_{i,N}(\theta) + \rho_i H_i \tau_{iS}(\theta_{EFF})$$
(38)

The method for calculating the effective downward flux of radiation $R_i \downarrow$ is channel dependent, so there are three expressions to be written in finite-difference form. For the microwave channels, the finite-difference representation of (11) is

$$R_i \downarrow = T_{BB}\tau_{i,N}(\theta) + \tau_{i,N}(\theta) \sum_{l=1}^{N} \bar{T}_l \left(\frac{1}{\tau_{i,l}(\theta)} - \frac{1}{\tau_{i,l-1}(\theta)} \right)$$
(39)

For IR channels where the atmosphere is optically thin, we follow (7) and write

$$R_i \downarrow = 2 \cos \theta \sum_{l=1}^{N} B_i(\bar{T}_l) \left(\tau_{i,l-1}(\theta) - \tau_{i,l}(\theta) \right)$$

$$\tag{40}$$

Other IR channels use the modeled downward flux as in (8)

$$R_i \downarrow = F_i B_i(T_s) (1 - \tau_{iN}(\theta)) \tag{41}$$

where T_s is the surface air temperature.

For the solar contribution term, the bi-directional reflectance ρ_i is taken as an input parameter since it (along with cloud-cleared radiances) is available from the level 2 Pathfinder data sets. The solar radiation H_i is

$$H_i = \pi \left(\frac{r_{\text{sun}}}{R_e}\right)^2 \cos(\varphi) B_i(5600 \text{K}) \tag{42}$$

where r_{sun} is the Sun's radius, R_e is the radius of Earth's orbit and φ is the solar zenith angle.

3.2 Finite-difference Jacobian calculation

The calculation of the Jacobian, or derivative of brightness temperature with respect to input parameters is accomplished through operation on the finite-difference representation of the forward problem as expressed in the Fortran code. For our current work, we are only interested in the dependence of brightness temperature on changes in the atmospheric profiles of temperature, moisture and ozone, so derivatives with respect to other parameters (e.g. surface and skin temperature, surface emissivity, reflectance, etc.) will not be considered.

This discussion begins with derivation of a few general expressions needed for the Jacobian calculation. Then, the derivative of the atmospheric term (vertical summation) with respect to ozone and water vapor is obtained. The derivative for temperature is discussed separately since its form is different from the ozone and water vapor derivative. We finish by discussing the derivative of the reflected radiance term, which makes use of terms derived in earlier parts of this section.

General expressions The expressions needed for the Jacobian calculation can be derived by repeated applications of the chain rule for derivatives. Brightness temperature is defined as $\Theta_i = B^{-1}(R_i)$, where B^{-1} is the inverse Planck function. Thus, for some input quantity Q:

$$\frac{\partial \Theta}{\partial Q} = \frac{\partial \Theta_{i}}{\partial R_{i}} \frac{\partial R_{i}}{\partial Q} \tag{43}$$

$$= \frac{\partial \Theta_{i}}{\partial R_{i}} \frac{\partial}{\partial Q} \left[\varepsilon_{i}(\theta) B_{i}(T_{g}) \tau_{i,N}(\theta) + \sum_{l=1}^{N} B_{i}(\bar{T}_{l}) \left(\tau_{i,l-1}(\theta) - \tau_{i,l}(\theta) \right) + R_{i}'(\theta) \right] \tag{44}$$

$$= \frac{\partial \Theta_{i}}{\partial R_{i}} \left[\frac{\partial}{\partial Q} \left\{ \varepsilon_{i}(\theta) B_{i}(T_{g}) \tau_{i,N}(\theta) \right\} + \frac{\partial}{\partial Q} \sum_{l=1}^{N} B_{i}(\bar{T}_{l}) \left(\tau_{i,l-1}(\theta) - \tau_{i,l}(\theta) \right) + \frac{\partial}{\partial Q} R_{i}'(\theta) \right] \tag{45}$$

The derivative of the Planck function with respect to temperature is

$$\frac{\partial B(T)}{\partial T} = \frac{2h\nu^3}{c^2(e^{\frac{h\nu}{kT}} - 1)^2} e^{\frac{h\nu}{kT}} \frac{h\nu}{kT^2}$$
 (46)

At brightness temperature Θ_i we can write the derivative $\partial \Theta_i/\partial R_i$ as

$$\frac{\partial \Theta_i}{\partial R_i} = \frac{\partial}{\partial R} B^{-1}(R_i) = \left[\frac{\partial B_i(T)}{\partial T} \right]_{T=\Theta}^{-1}$$
(47)

Water vapor and ozone derivatives The next step in obtaining the Jacobian is to find the derivative of the term in the vertical sum, which is the product of brightness temperature and transmittance. If Q_m is water vapor or ozone at a particular level m, we need only consider the transmittance derivatives. From (45) we may write:

$$\frac{\partial \Theta_i}{\partial Q_m} = \frac{\partial \Theta_i}{\partial R_i} \left[\varepsilon_i B_i(T_g) \frac{\partial \tau_{i,N}(\theta)}{\partial Q_m} + \sum_{l=1}^N B_i(\bar{T}_l) \frac{\partial}{\partial Q_m} (\tau_{i,l-1}(\theta) - \tau_{i,l}(\theta)) + \frac{\partial R_i'}{\partial Q_m} \right]$$
(48)

From (35) we may write (from the product rule or by logarithmic differentiation)

$$\frac{\partial \tau_{i,l}(\theta)}{\partial Q_m} = \tau_{a_j}(\theta) \frac{\partial}{\partial Q_m} \prod_{j=1}^l \tilde{\tau}_{i,j}(\theta)$$

$$= \tau_{i,l}(\theta) \sum_{i=1}^l \frac{1}{\tilde{\tau}_{i,j}(\theta)} \frac{\partial \tilde{\tau}_{i,j}(\theta)}{\partial Q_m} \tag{49}$$

Recall that the layer transmittance $\tilde{\tau}_{i,j}(\theta)$ is obtained by linear interpolation from transmittances calculated at rapid algorithm angles; thus, we have:

$$\frac{\partial \tilde{\tau}_{i,j}(\theta)}{\partial Q_m} = (1 - \alpha) \frac{\partial \tilde{\tau}_{i,j,k}}{\partial Q_m} + \alpha \frac{\partial \tilde{\tau}_{i,j,k+1}}{\partial Q_m}$$
 (50)

where θ_k and θ_{k+1} are the rapid algorithm zenith angles which bracket the desired angle θ and $\alpha = (\sec(\theta) - \sec(\theta_k))/(\sec(\theta_{k+1}) - \sec(\theta_k))$. Substituting into (49), we have

$$\frac{\partial \tau_{i,l}(\theta)}{\partial Q_m} = \tau_{i,l}(\theta) \sum_{j=1}^{l} \frac{1}{\tilde{\tau}_{i,j}(\theta)} \left\{ (1 - \alpha) \frac{\partial \tilde{\tau}_{i,j,k}}{\partial Q_m} + \alpha \frac{\partial \tilde{\tau}_{i,j,k+1}}{\partial Q_m} \right\}$$
(51)

Let's concentrate now on a specific quantity such as ozone column density for layer m $(Q_m = u_m)$. We know that

$$\tilde{\tau}_{i,j,k} = \tilde{\tau}_{F_{i,j,k}} \tilde{\tau}_{O_{i,j,k}} \tilde{\tau}_{W_{i,j,k}} \tag{52}$$

Since by definition $\tilde{\tau}_{F_i}$ and $\tilde{\tau}_{W_i}$ do not depend on ozone we need only consider $\tilde{\tau}_{O_i}$ when calculating the derivative of $\tilde{\tau}_{i,j,k}$. We can write:

$$\frac{\partial \tilde{\tau}_{O_{i,j,k}}}{\partial u_m} = -\tilde{\tau}_{O_{i,j,k}} F_{i,j,k} [1 - G_{i,k}(\bar{T}_j - 273)] N_i u_j^{N_i - 1} \frac{\partial u_j}{\partial u_m}
= \frac{\partial \tilde{\tau}_{O_{i,m}}}{\partial u_m} \delta_{jm}$$
(53)

since the ozone column density in layer m enters into the calculation of the effective layer transmittance only at layer m. We can write similar expressions for the derivatives with respect to specific humidity. The derivative $\partial \tau_{W_{i,j,k}}/\partial q_m$ is almost the same as $\partial \tau_{O_{i,j,k}}/\partial u_m$ but with an added term for the water vapor continuum.

Then we can also write:

$$\frac{\partial \tilde{\tau}_{i,j,k}}{\partial u_m} = \tilde{\tau}_{F_{i,m,k}} \tilde{\tau}_{W_{i,m,k}} \frac{\partial \tilde{\tau}_{O_{i,m,k}}}{\partial u_m} \delta_{jm}
= \frac{\tilde{\tau}_{i,m,k}}{\tilde{\tau}_{O_{i,m,k}}} \frac{\partial \tilde{\tau}_{O_{i,m,k}}}{\partial u_m} \delta_{jm}
\frac{\partial \tilde{\tau}_{i,j,k}}{\partial q_m} = \tilde{\tau}_{F_{i,m,k}} \tilde{\tau}_{O_{i,m,k}} \frac{\partial \tilde{\tau}_{W_{i,m,k}}}{\partial u_m} \delta_{jm}
= \frac{\tilde{\tau}_{i,m,k}}{\tilde{\tau}_{W_{i,m,k}}} \frac{\partial \tilde{\tau}_{W_{i,m,k}}}{\partial u_m} \delta_{jm}$$
(55)

Since $\partial \tilde{\tau}_{i,j,k}/\partial Q_m = 0$ for $j \neq m$ (for $Q_m = q_m$ or u_m) we may also drop the summation over j in (49) and (51) and write, for l > m

$$\frac{\partial \tau_{i,l}(\theta)}{\partial Q_m} = \frac{\tau_{i,l}(\theta)}{\tilde{\tau}_{i,m}(\theta)} \frac{\partial \tilde{\tau}_{i,m}(\theta)}{\partial Q_m} = \frac{\tau_{i,l}(\theta)}{\tilde{\tau}_{i,m}(\theta)} \left\{ (1 - \alpha) \frac{\partial \tilde{\tau}_{i,m,k}}{\partial Q_m} + \alpha \frac{\partial \tilde{\tau}_{i,m,k+1}}{\partial Q_m} \right\}$$
(56)

$$\frac{\partial}{\partial Q_m} (\tau_{i,l-1}(\theta) - \tau_{i,l}(\theta)) = \frac{(\tau_{i,l-1}(\theta) - \tau_{i,l}(\theta))}{\tilde{\tau}_{i,m}(\theta)} \frac{\partial \tilde{\tau}_{i,m}(\theta)}{\partial Q_m} \\
= \frac{(\tau_{i,l-1}(\theta) - \tau_{i,l}(\theta))}{\tilde{\tau}_{i,m}(\theta)} \left\{ (1 - \alpha) \frac{\partial \tilde{\tau}_{i,m,k}}{\partial Q_m} + \alpha \frac{\partial \tilde{\tau}_{i,m,k}}{\partial Q_m} \right\}$$
(57)

These expressions can be substituted into (48) to compute the ozone and water vapor Jacobians.

$$\frac{\partial \Theta}{\partial Q_m} = \frac{\partial \Theta}{\partial R} \left(\frac{1}{\tilde{\tau}_{i,m}(\theta)} \left\{ (1 - \alpha) \frac{\partial \tilde{\tau}_{i,m,k}}{\partial Q_m} + \alpha \frac{\partial \tilde{\tau}_{i,m,k+1}}{\partial Q_m} \right\} \right) \\
\left[\varepsilon_i B_i(T_g) \tau_{i,N}(\theta) - B_i(\bar{T}_m) \tau_{i,m}(\theta) + \sum_{l=m+1}^N B_i(\bar{T}_l) (\tau_{i,l-1}(\theta) - \tau_{i,l}(\theta)) \right] + \frac{\partial R_i'}{\partial Q_m} \right) (58)$$

Temperature derivative For the temperature derivative, there are additional terms for the derivative of the Planck function. For convenience, we calculate the derivative with respect to mean layer temperature $\bar{T}_m = 0.5(T_{m-1} + T_m)$

$$\frac{\partial \Theta_{i}}{\partial \bar{T}_{m}} = \frac{\partial \Theta_{i}}{\partial R_{i}} \left[\varepsilon_{i} B_{i}(T_{g}) \frac{\partial \tau_{iN}}{\partial \bar{T}_{m}} + \frac{\partial B_{i}(\bar{T}_{m})}{\partial \bar{T}_{m}} (\tau_{i,m-1} - \tau_{i,m}) + \sum_{l=1}^{N} B_{i}(\bar{T}_{l}) \frac{\partial}{\partial \bar{T}_{m}} (\tau_{i,l-1} - \tau_{i,l}) + \frac{\partial R_{i}'}{\partial \bar{T}_{l}} \right]$$
(59)

and then use that derivative in calculating the level temperature Jacobian

$$\frac{\partial \Theta_i}{\partial T_j} = \sum_{m=0}^{N} \frac{\partial \Theta_i}{\partial \bar{T}_m} \frac{\partial \bar{T}_m}{\partial T_j} = 0.5 \left[\frac{\partial \Theta}{\partial \bar{T}_{j-1}} + \frac{\partial \Theta}{\partial \bar{T}_j} \right]$$
(60)

The temperature derivative has terms for the derivative of the Planck function and the derivative of the transmittances with respect to temperature. In practice, the transmittance derivative is often neglected because it is assumed to be much smaller than the derivative of the Planck function. The Planck function derivative was given in (46). The derivation of the transmittance derivative is similar to that in the previous section, up to (51). Unlike ozone and water vapor, there is a temperature contribution in all three components $(\tau_{F_i}, \tau_{O_i}, \tau_{W_i})$ of the effective mean layer transmittance. Thus, by logarithmic differentiation we write:

$$\frac{\partial \tilde{\tau}_{i,j,k}}{\partial \bar{T}_m} = \tilde{\tau}_{i,j,k} \left(\frac{1}{\tilde{\tau}_{F_{i,j,k}}} \frac{\partial \tilde{\tau}_{F_{i,j,k}}}{\partial \bar{T}_m} + \frac{1}{\tilde{\tau}_{O_{i,j,k}}} \frac{\partial \tilde{\tau}_{O_{i,j,k}}}{\partial \bar{T}_m} + \frac{1}{\tilde{\tau}_{W_{i,j,k}}} \frac{\partial \tilde{\tau}_{W_{i,j,k}}}{\partial \bar{T}_m} \right)$$
(61)

The derivatives of the water vapor and ozone contributions to transmittance with respect to temperature are relatively simple:

$$\frac{\partial \tilde{\tau}_{W_{i,j,k}}}{\partial \bar{T}_m} = \left[D_{i,j,k} E_{i,k} (\bar{T}_j - 273) u_{q_j}^{M_i} - \sec(\theta_k) \frac{\partial K_{H_2 O_{ij}}}{\partial \bar{T}_m} \right] \tau_{W_{i,j,k}} \delta_{jm}$$
(62)

$$\frac{\partial \tilde{\tau}_{O_{i,j,k}}}{\partial \bar{T}_m} = \left(F_{i,j,k} G_{i,k} (\bar{T}_j - 273) u_{O_3 j}^{N_i} \right) \tau_{O_{i,j,k}} \delta_{jm} \tag{63}$$

For these derivatives (as with the transmittance derivatives with respect to ozone and water vapor) the mean temperature for a given layer affects only the effective transmittance contribution for that same layer.

The temperature derivative of the fixed gas contribution is complicated by the effective mean temperature term. Since we calculate effective mean temperature for a layer through a weighted sum of temperatures from all layers above, the temperature in a given layer affects the effective mean layer transmittance for every layer below that one.

$$\frac{\partial \tilde{\tau}_{F_{i,j,k}}}{\partial \bar{T}_m} = B_{i,j,k} \frac{\partial \bar{T}_j}{\partial \bar{T}_m} + C_{i,j,k}^* \sum_{l=1}^j W_{i,l,k} \frac{\partial \bar{T}_l}{\partial \bar{T}_j}$$

$$= B_{i,j,k} \delta_{jm} + C_{i,j,k}^* \sum_{l=1}^j W_{i,l,k} \delta_{lm} \tag{64}$$

Thus

$$\frac{\partial \tilde{\tau}_{F_{i,j,k}}}{\partial \bar{T}_m} = \begin{cases}
0 & j < m \\
B_{i,j,k} + C_{i,j,k}^* W_{i,m,k} & j = m \\
C_{i,j,k}^* W_{i,m,k} & j > m
\end{cases}$$
(65)

and

$$\frac{\partial \tilde{\tau}_{i,j,k}}{\partial \bar{T}_{m}} = \begin{cases}
0 & j < m \\
\tilde{\tau}_{i,j,k} \left(\frac{1}{\tilde{\tau}_{F_{i,j,k}}} \frac{\partial \tilde{\tau}_{F_{i,j,k}}}{\partial \bar{T}_{m}} + \frac{1}{\tilde{\tau}_{O_{i,j,k}}} \frac{\partial \tilde{\tau}_{O_{i,j,k}}}{\partial \bar{T}_{m}} + \frac{1}{\tilde{\tau}_{W_{i,j,k}}} \frac{\partial \tilde{\tau}_{W_{i,j,k}}}{\partial \bar{T}_{m}} \right) & j = m \\
\frac{\tilde{\tau}_{i,j,k}}{\tilde{\tau}_{F_{i,j,k}}} \frac{\partial \tilde{\tau}_{F_{i,j,k}}}{\partial \bar{T}_{m}} & j > m
\end{cases}$$
(66)

Also

$$\frac{\partial \tilde{\tau}_{i,j}(\theta)}{\partial \bar{T}_m} = \begin{cases} 0 & j < m \\ (1 - \alpha) \frac{\partial \tilde{\tau}_{i,j,k}}{\partial \bar{T}_m} + \alpha \frac{\partial \tilde{\tau}_{i,j,k+1}}{\partial \bar{T}_m} & j \ge m \end{cases}$$
(67)

and since the temperature in a given layer only affects the effective mean layer transmittance in layers at and below that layer we can write for $l \ge m$

$$\frac{\partial \tau_{i,l}(\theta)}{\partial \bar{T}_m} = \tau_{i,l}(\theta) \sum_{i=m}^{l} \frac{1}{\tilde{\tau}_{i,j}(\theta)} \frac{\partial \tilde{\tau}_{i,j}(\theta)}{\partial \bar{T}_m}$$
(68)

Since $\partial \tau_{i,l}(\theta)/\partial \bar{T}_m = 0$ for l < m we can also rewrite (59) to get:

$$\frac{\partial \Theta_{i}}{\partial \bar{T}_{m}} = \frac{\partial \Theta_{i}}{\partial R_{i}} \left[\varepsilon_{i} B_{i}(T_{g}) \frac{\partial \tau_{iN}}{\partial \bar{T}_{m}} + \frac{\partial B_{i}(\bar{T}_{m})}{\partial \bar{T}_{m}} (\tau_{i,m-1} - \tau_{i,m}) - B_{i}(\bar{T}_{m}) \frac{\partial \tau_{i,m}}{\partial \bar{T}_{m}} \right. \\
+ \sum_{l=m+1}^{N} B_{i}(\bar{T}_{l}) \left(\frac{\partial \tau_{i,l-1}}{\partial \bar{T}_{m}} - \frac{\partial \tau_{i,l}}{\partial \bar{T}_{m}} \right) + \frac{\partial R_{i}'}{\partial \bar{T}_{l}} \right]$$
(69)

Derivative of reflected radiance term Obtaining the derivatives of the reflected radiance term is the final step in the calculation of the Jacobians. From (38) we can write:

$$\frac{\partial R_i'(\theta)}{\partial Q} = (1 - \varepsilon_i) \frac{\partial R_i \downarrow}{\partial Q} \tau_{i,N}(\theta) + (1 - \varepsilon_i) R_i \downarrow \frac{\partial \tau_{i,N}}{\partial Q} + \rho_i H_i \frac{\partial \tau_{iS}(\theta_{EFF})}{\partial Q}$$
(70)

The derivatives of the transmittances $\partial \tau_{i,N}/\partial Q$ and $\partial \tau_{iS}(\theta_{EFF})/\partial Q$ can be obtained as in (56) and (68). This leaves us only with the derivative of the downward flux $\partial R_i \downarrow /\partial Q$ to consider.

The expressions for downward flux were given in (39)–(41). For microwave channels, the form of the downward flux derivative is the most complex. For the water vapor and ozone derivatives, we have:

$$\frac{\partial R_{i} \downarrow}{\partial Q_{m}} = \frac{\partial \tau_{i,N}(\theta)}{\partial Q_{m}} \left[T_{BB} + \sum_{l=1}^{N} \bar{T}_{l} \left(\frac{1}{\tau_{i,l}(\theta)} - \frac{1}{\tau_{i,l-1}(\theta)} \right) \right] + \frac{\tau_{i,N}(\theta)}{\tilde{\tau}_{i,m}(\theta)} \left\{ \frac{\bar{T}_{m}}{\tau_{i,m}(\theta)} + \sum_{l=m+1}^{N} \bar{T}_{l} \left[\left(\frac{1}{\tau_{i,l-1}(\theta)} \right) - \left(\frac{1}{\tau_{i,l}(\theta)} \right) \right] \right\} \frac{\partial \tilde{\tau}_{i,m}(\theta)}{\partial Q_{m}}$$
(71)

The expression for the temperature derivative is more complicated:

$$\frac{\partial R_{i} \downarrow}{\partial \bar{T}_{m}} = \frac{\partial \tau_{i,N}(\theta)}{\partial \bar{T}_{m}} \left[T_{BB} + \sum_{l=1}^{N} \bar{T}_{l} \left(\frac{1}{\tau_{i,l}(\theta)} - \frac{1}{\tau_{i,l-1}(\theta)} \right) \right]
+ \tau_{i,N}(\theta) \left[\left(\frac{1}{\tau_{i,m-1}(\theta)} \right) - \left(\frac{1}{\tau_{i,m}(\theta)} \right) \right] - \tau_{i,N}(\theta) \bar{T}_{m} \left(\frac{1}{\tau_{i,m}(\theta)} \right)^{2} \frac{\partial \tau_{i,m}(\theta)}{\partial \bar{T}_{m}}
+ \tau_{i,N}(\theta) \sum_{l=m+1}^{N} \bar{T}_{l} \left[\left(\frac{1}{\tau_{i,l-1}(\theta)} \right)^{2} \frac{\partial \tau_{i,l-1}(\theta)}{\partial \bar{T}_{m}} - \left(\frac{1}{\tau_{i,l}(\theta)} \right)^{2} \frac{\partial \tau_{i,l}(\theta)}{\partial \bar{T}_{m}} \right]$$
(72)

For IR channels where the atmosphere is optically thin, Eq. (40) for $R_i \downarrow$ involves a vertical sum—the atmospheric component of the radiance calculation for which we have already

derived (and in the modules have calculated) appropriate expressions. Thus we have

$$\frac{\partial R_i \downarrow}{\partial Q_m} = 2 \cos \theta \frac{\partial}{\partial Q_m} \sum_{l=m}^N B_i(\bar{T}_l) \left(\tau_{i,l-1}(\theta) - \tau_{i,l}(\theta) \right)$$
 (73)

For the other IR channels, the expression for the derivative of the downward flux is much simpler. The only term affected by changes in the atmospheric profile is the surface transmittance, thus:

$$\frac{\partial R_i \downarrow}{\partial Q_m} = -F_i B_i(T_s) \frac{\partial \tau_{iN}(\theta)}{\partial Q_m} \tag{74}$$

4 Modules for forward radiance calculation

The last two sections of this office note discuss the program modules used to implement the GLA forward model and Jacobian. This discussion is aimed primarily at the end user who may wish to call the forward model or Jacobian code from their program. The entry points to the main routines are given, along with a complete description of the input and output parameters. The program flow is documented by listing the subroutines called by the main routines and describing the calculations performed. Some relevant variables in the subroutines are described using mathematical formulas.

In this section, the Fortran modules used to implement the GLA forward model are discussed. The program flow is very simple to follow. The major steps are initialization of COMMON blocks with rapid algorithm coefficients, calculation of transmittances, and the forward model calculation of radiance (or brightness temperature). The routines responsible for these steps are initialGLA, tovs_tau, and hirsrad, respectively.

4.1 Initialization: initialGLA

Input parameters:

itrfdat INTEGER unit number (previously opened) of transmittance data file satelite CHARACTER*2 satellite identifier (see Table 2)

Output parameters:

bland LOGICAL (360,180) array of land/water flags
ierr INTEGER error flag for transmittance coefficients

0 successful read

1 error reading dataset

2 less than 24 channels found

Table 2: Satellite identifiers

$_{ m satellite}$	id
TIROS-N	TN
NOAA-6	NA
NOAA-7	NC
NOAA–8	NE
NOAA–9	NF
NOAA-10	NG
NOAA-11	NH
NOAA-12	ND

Subroutines called:

matcon	Sets math constants for retrievals
phycon	Sets physical constants for retrievals
grdcon	Sets vertical grid constants
chacon	Sets channel constants for TOVS channels
dattrf	Reads rapid algorithm transmittance coefficients

Before the transmittance and radiance calculations can be performed, coefficients and physical and mathematical constants must be loaded into the TOVS COMMON blocks. The initialGLA routine is an example driver for this; the input variables are the unit number of the transmittance coefficient file (previously opened by the main program) and a 2-letter character string indicating which satellite is being used. The current configuration only accommdates the use of coefficients from one satellite at a time. The initialGLA routine calls routines to initialize physical (phycon), mathematical (matcon) and vertical grid (grdcon) constants. This version of the initialGLA routine opens and reads in a dataset with land/water flags; this may not be necessary in future versions of the code if land/water flags are provided with the input profiles. After the land/water flags are set, routines are called to initialize channel-related constants (chacon) and to read in the rapid algorithm transmittance coefficients (dattrf). Each of the initialization routines phycon, matcon, grdcon, chacon and dattrf has an associated COMMON block defined in INCLUDE files phycons.h, matcons.h, grdcons.h, chacons.h and taucoef.h, respectively. Most of these initialization routines are straightforward, except for dattrf which is summarized below.

4.1.1 Transmittance coefficients: dattrf

The purpose of dattrf is to read transmittance coefficients, perform some calculations to convert the coefficients to the form which they will be used in the radiance calculation, and store the coefficients into the taucoef COMMON block. In the first step the zenith angles for the rapid algorithm coefficients are set up. Then, the coefficients for the rapid algorithm are read into temporary arrays. These coefficients were fitted to the original 66-level rapid algorithm; so the level-dependent coefficients at and above the 2 mb level are adjusted to fit the current 71-level vertical integration. In the final section of the routine, the coefficients are copied into the proper 'slots' in the common block, based on the input channel id.

In the last section of dattrf we also perform calculations to adjust the units of the transmittance coefficients to match those of the profiles used in the forward radiance calculation. The original transmittance coefficients were fitted for water vapor and ozone column densities expressed in terms of molecules/cm² while the current calculation uses densities of g/cm^2 . The conversion factors are raised to the same power as the column density (see (18)).

Some coefficients used in the fixed gas transmittance calculation are also set in the last section of dattrf. Eq. (26) showed how the coefficients associated with the fixed gas calculation could be arranged in a compact form; the calculations in the last section of dattrf are performed to obtain these new coefficients. The variables from dattrf associated with (26) are:

$$\begin{split} \mathbf{ZQ} &= 1 - \tau_{ij}^{\circ} = 1 - \prod_{l=1}^{j} A_{il}(\theta) \\ \mathbf{TAUCFW} &= \left(1 - A_{ik}(\theta)\right) \prod_{l=1}^{k-1} A_{il}(\theta) \\ \mathbf{TAUCFC} &= C_{ij}^{*} = \frac{C_{ij}(\theta)}{1 - \prod_{l=1}^{j} A_{il}(\theta)} = \frac{C_{ij}(\theta)}{1 - \tau_{ij}^{\circ}}. \end{split}$$

The dattrf routine was copied almost verbatim from the GLA TOVS retrieval code, to ensure the compatibility of the transmittance coefficients. Future versions of this code could be simplified if the coefficient database were reformatted, but the link to the original physics of the rapid algorithm would then be lost. Any revision of this routine should be done with care.

4.2 Transmittance calculation: tovs_tau

Input parameters:

tair	REAL (MAXLEV)	array of temperatures (K) at rapid algorithm levels
tmptop	REAL	temperature for top level of integration
h2omid	REAL (MAXLEV)	array of layer specific humidities (kg H_2O / kg air)
ozomid	REAL (MAXLEV)	array of layer ozone column densities (g / cm^2)
ps	REAL	surface pressure (mb)
nchan	INTEGER	number of channels to calculate
chanarr	INTEGER (MAXCHA)	array of channels to calculate; $1-20 = HIRS$, $21-24 = MSU$
nlev	INTEGER	index of surface level (also number of levels to use in calculation)
mw_zen	REAL	satellite zenith angle (degrees) for microwave channels
ir_zen	REAL	satellite zenith angle (degrees) for IR channels
dayflag	LOGICAL	true if "day" (sun angle > 85 degrees)
secsun	REAL	1. / cos(sun angle)

Output parameters:

tau REAL (MAXLEV, MAXCHA) transmittance along path to satellite

tausun REAL (MAXCHA) transmittance along path from sun to surface to satellite

errflag LOGICAL returns .true. if error in calculation

Subroutines called:

tovsfix_tau transmittance contribution due to fixed gases tovsh2o_tau transmittance contribution due to water vapor

tovsozo_tau transmittance contribution due to ozone

tovs_tauang interpolate layer transmittance to satellite zenith angle; take products over layers

The tovs_tau routine is the driver for the transmittance calculation. The input to this routine includes profiles of temperature at the rapid algorithm pressure levels and specific humidity and ozone column density for layers bounded by the pressure levels. The strategy used is to calculate variables common to all channel calculations (such as mean layer temperature, moisture variables, and zenith angle factors), then loop through the input channel list to calculate transmittances.

For the moisture variables, unit conversions are performed, since the input units of specific humidity (q: kg H₂O / kg air) are different from the units needed for the rapid algorithm (column density u_q : g cm⁻²) and the continuum model (vapor pressure e: atm). These conversions were discussed in (29) and (30).

$$\begin{split} \text{effh2o} &= \hat{u}_{H_2O_j} = \rho_{H_2O_j} \Delta z_j = \rho_{air} q_j \Delta z_j = \frac{10 \Delta p}{g} q_j \\ \text{e} &= e_j (\text{atm}) = \frac{\text{mol. air}}{\text{mol. H}_2O} \frac{\bar{p}(\text{mb})}{1013.25 \frac{\text{mb}}{\text{atm}}} q_j \end{split}$$

The routines tovsfix_tau, tovsh2o_tau and tovsozo_tau are called to calculate effective layer transmittances for the fixed gas contribution, $\tau_{iF}(p_l,\theta)$, water vapor contribution $\tau_{iW}(p_l,\theta)$, and ozone contribution $\tau_{iO}(p_l,\theta)$ (see Section 2.4 for discussion). The product of these effective layer transmittances calculated at the rapid algorithm angles is taken to obtain the layer transmittance at the rapid algorithm angles. These layer transmittances are interpolated linearly with respect to sec θ to the observed satellite zenith angle, and the product of the layer transmittances is taken to obtain the total transmittance between the satellite and each layer.

4.2.1 Fixed gas contribution: tovsfix_tau

In tovsfix_tau, the fixed gas contribution to effective layer transmittances is calculated using the model described in Section 2.4.2, with the coefficients modified as in Eq. (26).

The variables in tovsfix_tau associated with (26) are

$$\begin{array}{ll} \texttt{effTdif} &=& \sum_{k=1}^j W_{ik}(\theta)(\bar{T}_k - \bar{T}_k^\circ) \\ \\ \texttt{taufix} &=& \tilde{\tau}_{iF}(p_j, p_{j-1}, \theta) \end{array}$$

The effective temperature effTdif is carried as a running sum as the calculation of taufix is carried from the top of the atmosphere down to the surface.

4.2.2 Water vapor contribution: tovsh2o_tau

The contribution of water vapor to transmittance is calculated in tovsh2o_tau as a combination of the modeled water vapor transmittance $\tilde{\tau}_{iW}$ (18) and the water vapor continuum term K_{H_2O} (21).

Key variables in tovsh2o_tau include:

$$\begin{split} \operatorname{degchn} &= -\frac{dk}{dT} \\ &\operatorname{cont} = K_{H_2O_{ij}} = k_i \left(1 + \frac{dk}{dT} (\bar{T}_j - T_{cont,j}) \right) e_j \hat{u}_{H_2O_j} \\ &\operatorname{tauh2o} &= \hat{\tau}_{H_2O_{i,j,k}} = \exp \left[\left(-D_{i,j}(\theta) [1 - E_i(\bar{T}_j - 273)] u_{q_j}^{M_i} \right) - \sec(\theta) K_{H_2O_{ij}} \right] \end{split}$$

4.2.3 Ozone contribution: tovsozo_tau

The calculation of the ozone transmittance contribution is relatively simple. Since the input profile and transmittance coefficients are already expressed in terms of the same units, we need only perform the calculation for layer ozone transmittance contribution given in (18).

tauozo =
$$\tilde{\tau}_{O_{3i,j,k}} = \exp\left(-F_{i,j,k}[1 - G_{i,k}(\bar{T}_j - 273)]\{(\hat{u}_{O_{3j}})^{N_i}\}\right)$$

4.2.4 Zenith angle interpolation: tovs_tauang

After the product of the fixed gas, water vapor, and ozone contributions is taken to obtain the effective layer transmittance, a call is made to tovs_tauang to interpolate the layer transmittances to the proper zenith angle. The transmittance between each rapid algorithm level and the satellite is calculated as the product of all the layer transmittances above that level (12). For the IR channels, the transmittances are multiplied by the scattering factor described in Section 2.4.4. Finally, if solar effects will need to be calculated (i.e. daytime observation for IR channels 13–19), the transmittance is also interpolated to the effective solar zenith angle

$$\theta_{EFF} = \sec^{-1}(\sec(\varphi) + \sec(\theta))$$

(where φ is the solar zenith angle) along with the appropriate scattering factors.

Some relevant variables are

$$\begin{aligned} \text{tauang} &= \tilde{\tau}_{i,j,k} = \tilde{\tau}_{F_{i,j,k}} \tilde{\tau}_{H_2O_{i,j,k}} \tilde{\tau}_{O_{3i,j,k}} \\ \text{tau} &= \tau_{i,j}(\theta) = \tau_{a_{i,j}}(\theta) \prod_{l=1}^j \left\{ \tilde{\tau}_{i,l,k} + \frac{\sec(\theta) - \sec(\theta_k)}{\sec(\theta_{k+1}) - \sec(\theta_k)} (\tilde{\tau}_{i,l,k+1} - \tilde{\tau}_{i,l,k}) \right\} \end{aligned}$$

4.3 Radiance calculation: hirsrad

Input parameters:

tau	DEAT (MAYIEV MAYCHA)	transmittance along path to satellite
tausun	REAL (MAXCHA)	transmittance along path from sun to surface to satellite
nchan	INTEGER	number of channels to calculate
chanarr	INTEGER (MAXCHA)	array of channels to calculate; $1-20 = HIRS$, $21-24 = MSU$
tair	REAL (MAXLEV)	array of temperatures (K) at rapid algorithm levels
tmptop	REAL	temperature for top level of integration
ir_zen	REAL	satellite zenith angle (degrees) for IR channels
emissmw	REAL	microwave surface emissivity
land	LOGICAL	land/water flag (.true. if over land)
nlev	INTEGER	index of surface level (also number of levels to use in calculation)
rho	REAL	bidirectional surface reflectance
sunzang	REAL	solar zenith angle (degrees)
dayflag	LOGICAL	flag for solar calculation, .true. if day
tground	REAL	skin (ground) temperature (K)
tsurfair	REAL	surface air temperature (K)

Output parameters:

calc_rad REAL (MAXCHA)

IR channels: calculated radiance (w/ cm² sr)

MW channels: calculated brightness temperature (K)

The forward radiance calculation is carried out as described in Sections 2.1 and 3. The vertical sums are carried out separately for infrared and microwave channels, with surface terms and reflected radiance added afterwards.

The contribution from reflected radiance is calculated as described in Section 2.3, according to (38)–(42). For channels 8, 10, 18, and 19 (use_refl = .true.) where (40) applies, the atmospheric integral part of $R_i \downarrow$ is simply the partial sum in crad at that point (since surface terms have not yet been added). The solar contribution (second term in (38)) is only used for daytime calculations for IR channels 13–19.

5 Modules for Jacobian calculation

5.1 Brightness temperature Jacobian: tovs_tauK

Input parameters:

tair	REAL (MAXLEV)	array of temperatures (K) at rapid algorithm levels
tmptop	REAL	temperature for top level of integration
h2omid	REAL (MAXLEV)	array of layer specific humidities (kg H_2O / kg air)
ozomid	REAL (MAXLEV)	array of layer ozone column densities (g / cm^2)
ps	REAL	surface pressure (mb)
nchan	INTEGER	number of channels to calculate
chanarr	INTEGER (MAXCHA)	array of channels to calculate; $1-20 = HIRS$, $21-24 = MSU$
nlev	INTEGER	index of surface level (also number of levels to use in calculation)
mw_zen	REAL	satellite zenith angle (degrees) for microwave channels
ir_zen	REAL	satellite zenith angle (degrees) for IR channels
dayflag	LOGICAL	true if "day" (sun angle > 85 degrees)
secsun	REAL	1. / cos(sun angle)
emissmw	REAL	microwave surface emissivity
land	LOGICAL	land/water flagtrue. if measurement over land
rho	REAL	bi-directional surface reflectance
tground	REAL	skin (ground) temperature (K)
tsurfair	REAL	surface air temperature (K)

Output parameters:

tau	REAL (MAXLEV, MAXCHA)	transmittance along path to satellite
tausun	REAL (MAXCHA)	transmittance along path from sun to surface to satellite
bt	REAL (MAXCHA)	brightness temperatures (K)
dTbdT	REAL (MAXLEV, MAXCHA)	temperature Jacobian $\partial \Theta_i/\partial T_j$
dTbdq	REAL (MAXLEV, MAXCHA)	water vapor Jacobian $\partial \Theta_i/\partial q_j$
dTbdu	REAL (MAXLEV, MAXCHA)	ozone Jacobian $\partial \Theta_i/\partial u_j$
errflag	LOGICAL	returns .true. if error in calculation

Subroutines called:

tovsfix_tauK	transmittance contrib. by fixed gases and derivative
tovsh2o_tauK	transmittance contrib. by water vapor and derivative
tovsozo_tauK	transmittance contrib. by ozone and derivative
tovs_tauprodK	take product of fixed, water vapor and ozone contributions and derivatives
tovs_tauangK	interpolate transmittance to obs. zenith angle and calculate consistent derivative terms
dTb_dTQU	calculate brightness temperature and Jacobian

The calculation of the brightness temperature Jacobian follows essentially the same steps as forward problem calculation of radiance. (Actually, transmittances are set up and the forward radiance calculation is performed during the process of obtaining the Jacobian.) The mapping between forward problem modules and Jacobian modules is indicated by adding the suffix K to the module name, *i.e.* tovsozo_tauK is the Jacobian corresponding to tovsozo_tau in the forward model.

The finite-difference representations of the equations used were given in Section 3.2. The strategy for the Jacobian calculation is to first calculate the derivatives of the effective layer transmittance contributions from fixed gases, water vapor, and ozone (in tovsfix_tauK, tovsh2o_tauK and tovsozo_tauK). These derivatives are combined to obtain the derivatives of effective layer transmittance (tovs_tauprodK). The layer transmittance derivatives are interpolated to the appropriate zenith angle and combined to form the total transmittance derivatives (tovs_tauangK), which are used to calculate the brightness temperature Jacobians (dTb_dTQU). Since many of the transmittance terms involve calculation of products, it is convenient to calculate and store the logarithmic derivative of such terms (i.e. $\frac{1}{F} \frac{\partial F}{\partial Q}$ rather than $\frac{\partial F}{\partial Q}$) since, if F = abc, then by the product rule

$$\frac{\partial F}{\partial Q} = \frac{\partial (abc)}{\partial Q} = F\left(\frac{1}{a}\frac{\partial a}{\partial Q} + \frac{1}{b}\frac{\partial b}{\partial Q} + \frac{1}{c}\frac{\partial c}{\partial Q}\right)$$

Another thing to note about the derivatives is that when the transmittance is set to zero, we also set the derivative to zero. The GLA retrieval code (and also this forward radiance code) contains tests to prevent negative transmittance values; if a transmittance goes negative it is set to zero. Then, since the transmittance has been set to a constant value, its derivative must also be zero.

5.1.1 Fixed gas contribution: tovsfix_tauK

The fixed gas contribution to transmittance is a function only of temperature. We can use the derivative formula (65) to obtain the logarithmic derivative

$$\mathtt{dttaufix} = \frac{1}{\tilde{\tau}_{F_{i,j,k}}} \frac{\partial \tilde{\tau}_{F_{i,j,k}}}{\partial \bar{T}_m} = \left\{ \begin{array}{ll} 0 & j < m \\ \frac{1}{\tilde{\tau}_{F_{i,j,k}}} \left(B_{i,j,k} + C^*_{i,j,k} W_{i,m,k}\right) & j = m \\ \frac{C^*_{i,j,k}}{\tilde{\tau}_{F_{i,j,k}}} W_{i,m,k} & j > m \end{array} \right.$$

Since this derivative is zero for transmittance layers j < m, it is efficient to use a packed matrix to store dttaufix. (In fact, the time required to calculate the Jacobian was reduced about 20 % when packed notation is used compared with a full matrix representation.) If we take dttaufix = $\frac{1}{\bar{\tau}_{F_{i,j,k}}} \frac{\partial \tilde{\tau}_{F_{i,j,k}}}{\partial T_m} = a_{mj}$ then the matrix has the form

$$\mathtt{dttaufix} = \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} & \dots & a_{1N} \\ 0 & a_{22} & a_{23} & a_{24} & \dots & a_{2N} \\ 0 & 0 & a_{33} & a_{34} & \dots & a_{3N} \\ 0 & 0 & 0 & a_{44} & \dots & a_{4N} \\ \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & 0 & 0 & \dots & a_{NN} \end{bmatrix}$$

where we have taken the temperature layer as the first index and the transmittance layer as the second index. In the program, this upper triangular matrix is stored as

$$\mathtt{dttaufix} = [a_{11}, a_{12}, a_{22}, a_{13}, a_{23}, a_{33}, a_{14}, a_{24}, a_{34}, a_{44}, \dots]$$

and the index of (m, j) = (j - 1)(j/2) + m for 1 < m < j.

5.1.2 Water vapor contribution: tovsh2o_tauK

The derivatives of the water vapor and ozone contributions to effective layer transmittance are much simpler than the fixed gas term, since layer temperatures and column densities of ozone and water vapor only affect the layer transmittance for that same layer. (See (53) and (62)). Some expressions used in this module are:

$$\begin{split} \operatorname{celsius} &= \bar{T}_j - 273 \\ \operatorname{deffh2o} &= \frac{\partial u_{q_j}}{\partial q_m} = \frac{10 \Delta p_j}{g} \\ \operatorname{dqcont} &= \frac{\partial K_{H_2O_{ij}}}{\partial q_m} = 2k_i \left(1 + \frac{dk}{dT}(\bar{T}_j - T_{cont,j})\right) \frac{1}{\epsilon} \frac{\bar{p}(\operatorname{mb})}{1013.25 \frac{\operatorname{mb}}{\operatorname{atm}}} \frac{10 \Delta p}{g} q_m \delta_{jm} \\ \operatorname{dtcont} &= \frac{\partial K_{H_2O_{ij}}}{\partial \bar{T}_m} = k_i \frac{dk}{dT} \frac{1}{\epsilon} \frac{\bar{p}(\operatorname{mb})}{1013.25 \frac{\operatorname{mb}}{\operatorname{atm}}} \frac{10 \Delta p}{g} q_k^2 \delta_{jm} \\ \operatorname{dqtauh2o} &= \frac{1}{\tilde{\tau}_{H_2O_{i,j,k}}} \frac{\partial \tilde{\tau}_{H_2O_{i,j,k}}}{\partial q_m} \\ &= -D_{i,j,k} \{1 - E_{i,k}(\bar{T}_j - 273)\} M_i u_{q_j}^{M_i - 1} \delta_{jm} - \sec(\theta_k) \frac{\partial K_{H_2O_{ij}}}{\partial \bar{T}_m} \\ \operatorname{dttauh2o} &= \frac{1}{\tilde{\tau}_{H_2O_{i,j,k}}} \frac{\partial \tilde{\tau}_{H_2O_{i,j,k}}}{\partial \bar{T}_m} = D_{i,j,k} E_{i,k} u_{q_j}^{M_i} \delta_{jm} - \sec(\theta_k) \frac{\partial K_{H_2O_{im}}}{\partial \bar{T}_m} \end{split}$$

5.1.3 Ozone contribution: tovsozo_tauK

The calculation of the derivatives of the ozone contribution to mean layer transmittance is similar to that for the water vapor contribution. The derivatives calculated are:

$$\begin{split} \mathrm{dutauozo} &= \frac{1}{\tilde{\tau}_{O_{3i,j,k}}} \frac{\partial \tilde{\tau}_{O_{3i,j,k}}}{\partial q_m} = -F_{i,j,k} \{1 - G_{i,k}(\bar{T}_j - 273)\} N_i u_{O_{3j}}^{N_i - 1} \delta_{jm} \\ \mathrm{dttauozo} &= \frac{1}{\tilde{\tau}_{O_{3i,j,k}}} \frac{\partial \tilde{\tau}_{O_{3i,j,k}}}{\partial \bar{T}_m} = F_{i,j,k} G_{i,k} u_{O_{3j}}^{N_i} \delta_{jm} \end{split}$$

5.1.4 Transmittance product: tovs_tauprodK

In the modules for the forward radiance problem, the product of the effective layer transmittance contributions from fixed gases, water vapor and ozone is calculated in the main driver to obtain the layer transmittances. We calculate the derivative of that product in

this module. The derivatives for the layer transmittances were given in (54), (55) and (66). The relevant variables are

$$\begin{split} \mathrm{dqtauang} &= \frac{1}{\tilde{\tau}_{i,j,k}} \frac{\partial \tilde{\tau}_{i,j,k}}{\partial q_m} = \frac{1}{\tilde{\tau}_{W_{i,m,k}}} \quad \frac{\partial \tilde{\tau}_{W_{i,m,k}}}{\partial u_m} \delta_{jm} \\ \mathrm{dutauang} &= \frac{1}{\tilde{\tau}_{i,j,k}} \frac{\partial \tilde{\tau}_{i,j,k}}{\partial u_{O_{3m}}} = \frac{1}{\tilde{\tau}_{O_{i,m,k}}} \quad \frac{\partial \tilde{\tau}_{O_{i,m,k}}}{\partial u_m} \delta_{jm} \\ \mathrm{dttauang} &= \frac{1}{\tilde{\tau}_{i,j,k}} \frac{\partial \tilde{\tau}_{i,j,k}}{\partial \bar{T}_m} \\ &= \begin{cases} 0 & j < m \\ \frac{1}{\tilde{\tau}_{F_{i,j,k}}} \frac{\partial \tilde{\tau}_{F_{i,j,k}}}{\partial \bar{T}_m} + \frac{1}{\tilde{\tau}_{O_{i,j,k}}} \frac{\partial \tilde{\tau}_{O_{i,j,k}}}{\partial \bar{T}_m} + \frac{1}{\tilde{\tau}_{W_{i,j,k}}} \frac{\partial \tilde{\tau}_{W_{i,j,k}}}{\partial \bar{T}_m} & j = m \\ \frac{1}{\tilde{\tau}_{F_{i,j,k}}} \frac{\partial \tilde{\tau}_{F_{i,j,k}}}{\partial \bar{T}_m} & j > m \end{cases} \end{split}$$

5.1.5 Zenith angle interpolation: tovs_tauangK

In this routine the layer transmittances at the rapid algorithm zenith angles are interpolated to the desired zenith angle, and the product of the layer transmittances is taken to obtain the total transmittance between a given level and the satellite. The derivative of the interpolated layer transmittance is obtained by interpolating the derivatives of transmittance at the rapid algorithm angles. The derivative of the product of the layer transmittances is the sum of the logarithmic derivatives of layer transmittance. See equations (56) and (68). The variables used in the code are:

$$\begin{split} & \mathtt{tauint} = \frac{\partial \tilde{\tau}_{i,j}(\theta)}{\partial Q_m} = (1-\alpha) \frac{\partial \tilde{\tau}_{i,j,k}}{\partial Q_m} + \alpha \frac{\partial \tilde{\tau}_{i,j,k+1}}{\partial Q_m} \\ & \mathtt{dqtau} = \frac{1}{\tau_{i,l}(\theta)} \frac{\partial \tau_{i,l}(\theta)}{\partial q_m} = \frac{1}{\tilde{\tau}_{i,m}(\theta)} \left\{ (1-\alpha) \frac{\partial \tilde{\tau}_{i,m,k}}{\partial q_m} + \alpha \frac{\partial \tilde{\tau}_{i,m,k+1}}{\partial q_m} \right\} \\ & \mathtt{dutau} = \frac{1}{\tau_{i,l}(\theta)} \frac{\partial \tau_{i,l}(\theta)}{\partial u_m} = \frac{1}{\tilde{\tau}_{i,m}(\theta)} \left\{ (1-\alpha) \frac{\partial \tilde{\tau}_{i,m,k}}{\partial u_m} + \alpha \frac{\partial \tilde{\tau}_{i,m,k+1}}{\partial u_m} \right\} \\ & \mathtt{dttau} = \frac{1}{\tau_{i,l}(\theta)} \frac{\partial \tau_{i,l}(\theta)}{\partial \tilde{T}_m} = \sum_{j=m}^{l} \frac{1}{\tilde{\tau}_{i,j}(\theta)} \left\{ (1-\alpha) \frac{\partial \tilde{\tau}_{i,j,k}}{\partial \tilde{T}_m} + \alpha \frac{\partial \tilde{\tau}_{i,j,k+1}}{\partial \tilde{T}_m} \right\} \end{split}$$

5.1.6 Jacobian calculation: dTb_dTQU

The final step in the Jacobian calculation is to sum the transmittance derivatives (and for temperature, the derivatives of the Planck function) in the vertical and add derivatives of surface terms to obtain derivatives of radiance and brightness temperature with respect to the input temperature, water vapor, and ozone profiles. Relevant equations include (58) and (69), as well as the reflected radiance derivative terms discussed in (3.2)–(3.2).

The calculation of the vertical sums involves terms such as

dTb_dq(1) = dTb_dq(1) - btav * tau(1) * dqtau(1)
$$B_i(\bar{T}_l) \qquad \qquad \tau_{i,l}(\theta) \qquad \qquad \frac{1}{\tau_{i,l}(\theta)} \frac{\partial \tau_{i,l}(\theta)}{\partial q_l}$$

$$\label{eq:dtau} \begin{split} \mathrm{d} \mathrm{Tb} _\mathrm{d} \mathrm{Tav}(\mathbf{m}) &= \mathrm{d} \mathrm{Tb} _\mathrm{d} \mathrm{Tav}(\mathbf{m}) \; + \quad \mathrm{btav} \; * \quad (\; \mathrm{tau}(\mathbf{l} - \mathbf{1}) \; * \quad \mathrm{dttau}(\mathbf{m} + \mathbf{n} \mathbf{m} \mathbf{1}) \\ & B_i(\bar{T}_l) \qquad \qquad \tau_{i,l-1}(\theta) \qquad \frac{1}{\tau_{i,l-1}(\theta)} \frac{\partial \tau_{i,l-1}(\theta)}{\partial \bar{T}_m} \\ & - \quad \mathrm{tau}(\mathbf{l}) \; * \qquad \mathrm{dttau}(\mathbf{m} + \mathbf{n}) \;) \\ & \tau_{i,l}(\theta) \qquad \qquad \frac{\partial \tau_{i,l}(\theta)}{\partial \bar{T}_m} \end{split}$$

Remember that terms such as dttau and dqtau are logarithmic derivatives and thus have the derivative multiplied by $1/\tau$.

Acknowledgments

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A Forward radiance modules prologues and source code

Sample program

```
program sample
     implicit none
С
c sample program for GLA forward model and Jacobian
     include "tovsparam.h"
     integer
                                  ! loop counter
     character*80 trcoefs
                                  ! transmittance coef. file name
c inputs/outputs for initialGLA
                                  ! transmittance coef. unit number
     integer
                 itrfdat
     character*2 satelite
                                  ! satellite ID
                 bland(360,180) ! array of land/water flags
     logical
                                  ! error flag
     integer
                  ierr
c inputs for tovs_tau
c atmospheric values are at the 71 (maxlev) rapid algorithm pressure levels
              tair(maxlev)
                                  ! temperature (K)
     real
                                  ! temperature at "top of atmosphere"
     real
               tmptop
              h2omid(maxlev)
                                  ! layer specific humidity (kg/kg)
     real
     real
               ozomid(maxlev)
                                  ! ozone column density (g/cm**2)
     real
                                  ! surface pressure (mb)
              рs
     integer
                                  ! number of channels to calculate
              nchan
              chanarr(maxcha)
                                  ! array of channel numbers
     integer
     integer
              nlev
                                  ! number of levels in input sounding
                                 ! zenith angle for microwave channels
     real
              mw_zen
                                  ! zenith angle for infrared channels
     real
               ir_zen
                                  ! 'true' if daytime
     logical
               dayflag
     real
                                  ! 1. / cos( solar zenith angle )
               secsun
c outputs from tovs_tau, inputs to hirsrad
     real tau(maxlev, maxcha) ! transmittance along path to satellite
              tausun(maxcha)
                                ! transmittance along path from sun
     real
     logical errflag
                                  ! error return flag
c other inputs to hirsrad
     real
              emissmw
                                  ! microwave emissivity
                                  ! land/water flag
     logical
               land
     real
              rho
                                 ! bidirectional reflectance
                                 ! solar zenith angle
     real
              sunzang
     real
real
                                 ! surface skin temperature
              tground
                                 ! surface air temperature
               tsurfair
```

```
c output from hirsrad
              calc_rad(maxcha) ! calculated radiances
      real
c arrays for Jacobian calculation
      real dTbdt(maxlev,maxcha) ! deriv. wrt. input temperature real dTbdq(maxlev,maxcha) ! deriv. wrt. input specific humidity real dTbdu(maxlev,maxcha) ! deriv. wrt. input column density
      real dTbdt(maxlev,maxcha)
                                       ! deriv. wrt. input temperature
      real btnew(maxcha)
                                      ! calculated brightness temperature
      data itrfdat /23/
C.
c set satellite parameters, read in transmittance coefficient dataset
     (in initialGLA)
      satelite = 'NH'
      trcoefs = 'trcoefs8.nh.ieee'
      open(itrfdat,file=trcoefs,status='old',form='unformatted')
      call initialGLA(itrfdat,satelite,bland,ierr)
      close(itrfdat)
      if (ierr .ne. 0 ) then
         print *,' error returned from initialGLA',ierr
         stop
      endif
c input values for forward (and Jacobian) calculation
      print *,'input zenith angles to try'
      read(5,*) ir_zen,mw_zen
      print *,'input nchan (number of channels) and channel array'
      read(5,*) nchan,(chanarr(1),l=1,nchan)
      print *,'input number of levels in sounding, then T, H2O and O3'
      read(5,*) nlev
      do l = 1, nlev
         read(5,*) tair(1), h2omid(1), ozomid(1)
      enddo
      print *,'input surface pressure and top level temperature'
      read(5,*) ps, tmptop
      print *,'input surface air and ground (skin) temperature'
      read(5,*) tsurfair, tground
      print *,'input solar zenith angle, and land/water flag'
```

```
read(5,*) sunzang, land
 if (sunzang .lt. 89.) dayflag = .true.
 secsun = 1. / cos (sunzang * acos(0.) / 90.)
print *, 'input bidirectional reflectance and sfc. MW emissivity'
read(5,*) rho, emissmw
call tovs_tau(tair, tmptop, h2omid, ozomid, ps, nchan,
         chanarr, nlev, mw_zen, ir_zen, dayflag, secsun, tau,
&
         tausun, errflag)
call hirsrad(tau,tausun,nchan,chanarr,tair,tmptop,ir_zen,
&
         emissmw, land, nlev, rho, sunzang, dayflag, tground,
         tsurfair, calc_rad)
&
call tovs_tauK(tair, tmptop, h2omid, ozomid, ps, nchan,
     chanarr, nlev, mw_zen, ir_zen, dayflag, secsun,
      emissmw, land, rho, tground, tsurfair,
&
     tau, tausun, btnew, dTbdT, dTbdq, dTbdu, errflag)
stop
 end
```

Sample Makefile

```
# sample Makefile --- shows dependencies of files
INITIAL = chacon.o dattrf.o grdcon.o matcon.o phycon.o
TAUCALC = tovs_tau.o tovsfix_tau.o tovsh2o_tau.o tovsozo_tau.o tovs_tauang.o
MODULES = sunang.o hirsrad.o brtemp.o planck.o $(INITIAL) $(TAUCALC)
KMODULES = tovsh2o_tauK.o tovsozo_tauK.o tovsfix_tauK.o tovs_tauK.o \
                tovs_tauprodK.o tovs_tauangK.o dplanck.o dTb_dTQU.o
librad.a
                 : $(MODULES)
ar -r librad.a $(MODULES)
libradK.a
                 : $(KMODULES)
ar -r libradK.a $(KMODULES)
chacon.o
              : tovsparam.h chacons.h phycons.h
             : tovsparam.h chacons.h matcons.h phycons.h taucoef.h
dattrf.o
grdcon.o
             : tovsparam.h grdcons.h
hirsrad.o
              : tovsparam.h phycons.h chacons.h matcons.h
matcon.o
             : matcons.h
phycon.o
             : matcons.h phycons.h
sunang.o
             : phycons.h matcons.h
tovs_tau.o
             : tovsparam.h matcons.h grdcons.h chacons.h \
                 tovsfix_tau.o tovsh2o_tau.o tovsozo_tau.o tovs_tauang.o
tovs_tauang.o : tovsparam.h taucoef.h grdcons.h chacons.h
tovsfix_tau.o : tovsparam.h taucoef.h
tovsh2o_tau.o : tovsparam.h taucoef.h phycons.h chacons.h grdcons.h
tovsozo_tau.o : tovsparam.h taucoef.h
tovs_tauK.o : tovsparam.h matcons.h grdcons.h chacons.h taucoef.h \
                 tovsfix_tauK.o tovsh2o_tauK.o tovsozo_tauK.o \
                 tovs_tauangK.o
tovsfix_tauK.o : tovsparam.h taucoef.h
tovsh2o_tauK.o : tovsparam.h taucoef.h phycons.h chacons.h grdcons.h
tovsozo_tauK.o : tovsparam.h taucoef.h
tovs_tauprodK.o : tovsparam.h
                : tovsparam.h phycons.h chacons.h matcons.h \
dTb_dTQU.o
                  planck.o dplanck.o brtemp.o
```

"Include" files

Parameter definitions

tovsparam.h - parameter definitions

```
! beginning of include file tovsparam.h
!
! used in nearly all modules
!
! parameters for GLA TOVS rapid algorithm
!
   integer maxlev    ! maximum number of levels
   integer maxangs    ! maximum number of angles
   integer maxcha     ! maximum number of channels
   parameter( maxlev=71, maxangs=3, maxcha=24)
! end of include file tovsparam.h
```

Constants

chacons.h – channel constants

```
beginning of include file chacons.h
      initialized in: chacon
       used in: dattrf hirsrad tovs_tau tovs_tauang tovsh2o_tau
                       tovs_tauK tovsh2o_tauK dTb_dTQU
     channel constants used in GLA TOVS retrievals
1______
        character*8 chanid! channel identifier
        logical irchan
                                             ! true if ir channel
       logical mwchan ! true if mw channel
logical use_refl ! true if use integrated reflected IR in
! downward flux calculation
                      ! downward flux calculation
abscof ! absorp. coeff for H2O continuum
freq ! frequency (cm^-1 or ghz)
chscatt ! aerosol scattering & absorp. coeff.
contmp ! reference T for H2O continuum
degchn ! dK/dT for H2O continuum
dfxcon ! downward flux coefficient
em_land ! ir emissivity, land
em_water ! ir emissivity, water
planck1 ! const for planck function
planck2 ! const for planck function
        real
       real
        real
        real
        real
        real
        real
        real
        real
        real
```

common /chacons/ chanid(maxcha), irchan(maxcha), mwchan(maxcha)

```
&,use_refl(maxcha), abscof(maxcha), freq(maxcha), chscatt(maxcha)
     &,contmp(maxcha), degchn(maxcha), dfxcon(maxcha), em_land(maxcha)
     &,em_water(maxcha), planck1(maxcha), planck2(maxcha)
      end of include file chacons.h
grdcons.h – vertical grid constants
!
       beginning of include file grdcons.h
      initialized in: grdcon
       used in: tovs_tau tovs_tauang tovsh2o_tau
           tovs_tauK tovsh2o_tauK
      vertical grid constants used in GLA retrievals
   18May96 Meta S. added 'delprs' to common
1 -----
     real pres ! pressures at level boundaries for rapid alg.
real presa ! average layer pressures for rapid algorithm
real presln ! logarithm of pressure
real delprs ! layer difference in pressure * 10 / g
real scat ! coeff. for scattering calculation
real stntmp ! standard temperature profile
real tstd_av ! average layer temperature, std. profile
      common /grdcons/
     & pres( maxlev ), presa( maxlev ), presln( maxlev )
     & , delprs( maxlev), scat( maxlev ), stntmp( maxlev )
     & , tstd_av ( maxlev )
     end of include file grdcons.h
matcons.h - mathematical constants
      beginning of include file matcons.h
initialized in: matcon
   used in: dattrf hirsrad sunang tovs_tau tovs_tauang
              tovs_tauK dTb_dTQU
! mathematical constants as used in gla retrievals
deg2rad ! convert degrees to radians
pi ! pi
     real
     real
                                   рi
                  pid2 ! pi/2
     real
```

```
rad2deg
     real
                         ! convert radians to degrees
     real
                twopi
                           ! 2 * pi
     common /matcons/ deg2rad, pi, pid2, rad2deg, twopi
end of include file matcons.h
phycons.h – physical constants
     beginning of include file phycons.h
initialized in: phycon
   used in: chacon dattrf hirsrad sunang tovs_h2o_tau
            tovsh2o_tauK dTb_dTQU
   physical constants used in GLA retrievals
   18May96 Meta S. added 'gravcon', 'econ' to common
real
                avogad
                          ! Avogadro's number
               boltzmns ! Boltzmann's constant (joule, plancks ! Planck constant (joule sec) atmosph ! standard atmosphere (mb)
                          ! Boltzmann's constant (joule/K)
     real
     real
     real
                          ! Cosmic background bright. temp. (K)
     real
                bigbang
                grav
     real
                           ! acceleration from gravity (m/s)
    real
                          ! velocity of light (cm/s)
                clight
               daymaxd ! date of maximum solar declin. angle
     real
               days_per_yr ! no. of days per year (non-leap year)
     real
               declmax ! max solar declination angle
     real
                          ! planck funct. const #1 (see in
     real
               plcon1
     real
               plcon2
                          ! planck funct. const #2 phycon.f)
                          ! pi * (Rsun/Rearth-orbit)^2 steradians
     real
               scon
     real
                shmin
                          ! Minimum specific humid. for rapid alg
                          ! limiting solar zen angle for sun calc.
     real
               sunset
     real
               tsun
                          ! Sun temperature
               airmol
h2omol
ozomol
                          ! molecular weight of air
     real
     real
                          ! molecular weight of H2O
                          ! molecular weight of O3
     real
                          ! 10 / grav - used in thickness calc.
     real
                gravcon
                           ! (airmol/h2omol) /1013.25
     real
     common /phycons/ avogad, boltzmns, plancks, atmosph, bigbang
    & , grav , clight , daymaxd , days_per_yr
        , declmax , plcon1 , plcon2 , scon, shmin, sunset
        , tsun , airmol , h2omol , ozomol, gravcon, econ
```

end of include file phycons.h

!

taucoef.h – transmittance coefficients

```
beginning of include file taucoef.h
     initialized in: dattrf
     used in: tovs_tauang tovsfix_tau tovsh2o_tau tovsozo_tau
        tovs_tauK tovsfix_tauK tovsh2o_tauK tovsozo_tauK
    coefs for rapid algorithm computation of trans funs
    common /taucoef/
    $ taucfa (maxlev, maxangs, maxcha)
   $ , taucfb ( maxlev, maxangs, maxcha )
   $ , taucfc ( maxlev, maxangs, maxcha )
    $ , taucfd ( maxlev, maxangs, maxcha )
    $ , taucfe ( maxangs, maxcha )
    $ , taucff ( maxlev, maxangs, maxcha )
   $ , taucfg
                    ( maxangs, maxcha )
   $ , taucfm
                           ( maxcha )
                            ( maxcha )
    $ , taucfn
    $ , taucfw ( maxlev, maxangs, maxcha )
    $ , numangs
    $ , angle
                    ( maxangs )
   $ , secang
                   ( maxangs )
    double precision taucfa, taucfb, taucfc, taucfd, taucfe, taucff,
          taucfg, taucfm, taucfn, taucfw
    real angle
    real secang
    integer numangs
! tau_fixed = a + b (t - to) + c (t* - t*o)
! tau_h2o = exp(-(d(1-e(t-273))h2omid**m) + continuum term
! tau_ozone = exp( - (f(1 - g(t - 273)) ozomid**n)
end of include file taucoef.h
```

Fortran code

brtemp - Calculate brightness temperature

```
!!ROUTINE: brtemp
!!DESCRIPTION: COMPUTES BRIGHTNESS TEMPERATURE FROM RADIANCE VIA
           PLANCK FUNCTION. Derived from GLA retrieval code
! SYSTEM ROUTINES USED: none
! SUBROUTINES CALLED: none
!!INPUT PARAMETERS:
    RAD
          real RADIANCE FOR TEMPERATURE COMPUTATION
                UNITS WATTS/(CM**2-SR)
    PLANK1 real WAVE NUMBER DEPENDENT CONSTANT
                PLANK1 = 2 * VELLIGHT**2 * PLANCKCON / LAMDA**3
                PLANK1 = 1.193E-12/LAMDA**3 WATTS / (CM**2 - SR)
                  WHERE
                VELLIGHT = 3.00E+10 CM/SEC
                PLANKCON = 6.63E-34 JOULES-SEC
                      = WAVE LENGTH IN CM
                T. A MD A
    PLANK2 real WAVE NUMBER DEPENDENT CONSTANT
                PLANK2 = PLANCKCON*VELLIGHT / (BOLTZMAN*LAMDA)
                PLANK2 = 1.441/LAMDA
                                DEG.K
                  WHERE
                BOLTZMAN = 1.38E-23 JOULE / DEG.K
!!OUTPUT PARAMETERS:
          real Brightness temperature (K)
!!REVISION HISTORY:
    250CT84 JIMPF
                original program in GLA TOVS retrievals
    11DEC89 SYLEE
                UNVECTORIZED FULL PRECISION VERSION
    26SEP95 Meta S. converted to DAO standard (implicit none)
    100CT95 Meta S. revised prologue to DAO standard
subroutine brtemp (rad, plank1, plank2, bt)
    implicit none
    real
          bt
    real
          plank1
    real
          plank2
    real
           rad
! calculate brightness temperature
bt
              = plank1 / rad
```

chacon – Set channel constants

```
!!ROUTINE: chacon
!!DESCRIPTION: Set channel constants for TOVS (HIRS/MSU) channels
         Based on GLA TOVS retrieval code (chacon,
           addcha, and related subroutines)
! SYSTEM ROUTINES USED: none
! SUBROUTINES CALLED: none
!!INPUT PARAMETERS:
   satelite - character*2 identifier for satellite
!!OUTPUT PARAMETERS: none
!!REVISION HISTORY:
! 25sep95 Meta S. original routine derived from subroutines
            CHACON and ADDCHA in GLA retrieval code
            added central frequencies for NOAA-12 (ND)
 20feb96 Meta S.
            (Note, IR20 given dummy frequency - not listed
             in NOAA12 docs... wouldn't use IR20 anyway.)
! NOTES:
     will need some revision to restore multiple satellites
subroutine chacon (satelite)
implicit none
input variables
   character*2 satelite
                   ! identifier for satellite to use
include "tovsparam.h"
   include "chacons.h"
   include "phycons.h"
local variables
integer ic
   real tplcon1, tplcon2
```

```
! values for HIRS channels
! satellite dependent central frequencies of channels
                freqTN(20), freqNA(20), freqNC(20), freqNE(20) !frequency
       real
       real
                freqNF(20), freqNG(20), freqNH(20), freqND(20) ! cm^-1
      real
             freqHRS (20)
                                       ! frequency (cm^-1)
                                    ! absorp. coeff for H2O continuum
! dk/dT for H2O continuum
      real abscof_hrs(20)
      real degchn_hrs(20)
      real contemp_hrs(20) ! ref T for H2O continuum
real chscat_hrs(20) ! aerosol scattering & absorp. coeff.
real em_land_hrs(20) ! emissivity for land
      real em_water_hrs(20) ! emissivity for water
                                       ! downward flux coefficient
      real dfx_hrs(20)
! values for MSU channels
                                     ! frequency (GHz)
      real
             freqMSU(4)
      real abscof_msu(4) ! absorp. coeff for H2O continuum
real degchn_msu(4) ! dk/dT for H2O continuum
real contemp_msu(4) ! ref T for H2O continuum
real chscat_msu(4) ! aerosol scattering & absorp. coeff.
real em_land_msu(4) ! emissivity for land (not used?)
real em_water_msu(4) ! emissivity for water (not used?)
      real dfx_msu(4)
       character*6 chans(24) ! channel identifier
Ţ
                      HIRS/2 FREQUENCIES AND CHANNEL ID
                     TN(TIROS-N), NA(NOAA-6), NC(NOAA-7),
                     NE(NOAA-8), NF(NOAA-9), NG(NOAA-10),
                     NH(NOAA-11), ND(NOAA-12)
      DATA freqTN
                 / 668.70, 679.05, 689.70, 703.80, 716.70,
                    731.85, 749.50, 900.00, 1030.00, 1225.00, 1365.00, 1488.00, 2192.50, 2211.65, 2237.35,
                    2271.20, 2308.85, 2512.00, 2660.00,14500.00/
      DATA
               freqNA
                 / 667.95, 679.34, 689.99, 704.63, 717.75,
                     732.30, 749.16, 899.94, 1027.55, 1222.98,
                    1368.50, 1481.08, 2190.48, 2210.91, 2238.62,
                    2269.65, 2360.94, 2515.28, 2649.97,14500.00/
      DATA
               freqNC
                  / 668.70, 681.00, 692.00, 705.00, 718.00,
                     734.00, 753.00, 904.00, 1028.00, 1229.00,
                    1360.00, 1483.00, 2181.00, 2211.00, 2241.00,
                    2260.00, 2360.00, 2515.00, 2660.00,14500.00/
      DATA
               freqND
                 / 667.58, 680.18, 690.01, 704.22, 716.32,
                     732.81, 751.92, 900.45, 1026.66, 1223.44,
                    1368.68, 1478.59, 2190.37, 2210.51, 2236.62,
```

```
2267.62, 2361.64, 2514.68, 2653.48,14500.00/
     DATA
           freqNE
             / 667.41, 679.45, 690.90, 702.97, 717.56,
                732.97, 747.90, 901.08, 1027.11, 1224.05,
               1366.17, 1486.92, 2189.28, 2211.71, 2238.06,
               2271.43, 2357.11, 2515.53, 2661.85,14355.00/
     DATA
           freqNF
             / 667.67, 679.84, 691.46, 703.37, 717.16,
                732.64, 749.40, 898.53, 1031.61, 1224.74,
               1365.12, 1483.24, 2189.97, 2209.18, 2243.14,
               2276.46, 2359.05, 2518.14, 2667.80,14549.27/
     DATA
           freqNG
               667.70, 680.23, 691.15, 704.33, 716.30,
                733.13, 750.72, 899.50, 1029.01, 1224.07,
               1363.32, 1489.42, 2191.38, 2208.74, 2237.49,
               2269.09, 2360.00, 2514.58, 2665.38,14453.14/
     DATA
           freqNH
             / 668.99, 678.89, 689.70, 703.25, 716.83,
                732.11, 749.48, 900.51, 1031.19, 795.69,
               1361.10, 1479.86, 2189.94, 2209.66, 2239.26,
               2267.80, 2416.32, 2511.83, 2664.07,14453.14/
HIRS/2 CHANNEL CONSTANTS
     DATA
           abscof_hrs
    */30.32232, 28.83525, 27.34284, 25.63885, 24.13291,
    * 22.40358, 20.61069, 8.87156, 6.77743, 5.533819,
                           0.2740922, 0.2529629, 0.2265023,
    * 0.0,
                0.0,
                           0.06503328,0.06404681,0.0
    * 0.1949478, 0.0,
     DATA degchn_hrs
                            .02,
                                      .02,
    */ .02,
                .02,
                                               .02,
      .02,
                 .02,
                            .02,
                                      .02,
                                                .02,
                            .010404,
                                      .0103655, .01030173,
      .0,
                .0,
      .01019153, .009672762, .007432,
                                     .0055279, .0
     DATA
           contemp_hrs
             / 303.00,
                         303.00, 303.00,
                                          303.00,
                                                  303.00,
                         303.00, 303.00, 296.00,
                303.00,
                                                  296.00,
                  0.00,
                           0.00, 338.00, 338.00,
                                                  338.00,
                           0.00, 338.00, 338.00,
                338.00,
                                                    0.00/
     DATA
           chscat_hrs
                 1.00,
                           1.00,
                                   1.00,
                                            1.00,
                                                    1.00,
                                            1.00,
                  1.00,
                           1.00,
                                   1.00,
                                                    1.00,
                  1.00.
                           1.00.
                                   3.00.
                                            3.00.
                                                    3.00.
                           3.00,
                                   3.00,
                                            3.00,
                                                    0.00/
                  3.00,
     DATA
           em_land_hrs
             /
                 0.95,
                           0.95,
                                   0.95,
                                            0.95,
                                                    0.95,
                                            0.95,
                  0.95.
                           0.95,
                                   0.95,
                                                    0.90.
                                   0.85,
                                           0.85,
                  0.90,
                           0.90,
                                                    0.85,
                  0.85,
                           0.85,
                                   0.85,
                                           0.85,
                                                    0.00/
     DATA
            em_water_hrs
                          0.98,
                                 0.98,
                                         0.98,
                 0.98,
                                                    0.98,
```

```
0.98, 0.98, 0.98, 0.98, 0.98,
0.98, 0.98, 0.95, 0.95, 0.95,
0.95, 0.95, 0.95, 0.95,
                                                       0.00/
     DATA dfx_hrs
                   1.00, 1.00, 1.00, 1.00, 1.00, 0.71, 0.49, 1.00, 1.00, 1.00, 1.00, 1.00, 1.00, 1.00, 1.00, 1.00, 1.00, 0.95, 0.95, 1.00/
                                                      1.00/
MSU CHANNEL CONSTANTS
     DATA freqMSU / 50.30, 53.74, 54.96, 57.95 / DATA abscof_msu / 4*0.00 / DATA degchn_msu / 4*0.00 /
     DATA contemp_msu / 4*0.00 /
DATA chscat_msu / 4*0.00 /
DATA em_land_msu / 4*1.00 /
DATA em_water_msu / 4*1.00 /
     DATA dfx_msu / 4*0.95 /
     data chans / 'IR(01)', 'IR(02)', 'IR(03)', 'IR(04)', 'IR(05)',
     & 'IR(06)','IR(07)','IR(08)','IR(09)','IR(10)','IR(11)',
     % 'IR(12)','IR(13)','IR(14)','IR(15)','IR(16)','IR(17)','IR(18)',
     % 'IR(19)','IR(20)','MW(01)','MW(02)','MW(03)','MW(04)'/
! set up channel id's (used in identifying coefficients in 'dattrf')
1------
     do ic = 1,24
        write(chanid(ic),'(a2,a6)') satelite,chans(ic)
      enddo
SET THE INSTRUMENTS ( TOVS ONLY FOR NOW )
        IF ( satelite .EQ. 'TN' ) THEN
           DO ic = 1, 20
              freqHRS(ic) = freqTN(ic)
           enddo
        ELSE IF ( satelite .EQ. 'NA' ) THEN
           DO ic = 1, 20
              freqHRS(ic) = freqNA(ic)
        ELSE IF ( satelite .EQ. 'NC' ) THEN
           DO ic = 1, 20
              freqHRS(ic) = freqNC(ic)
        ELSE IF ( satelite .EQ. 'ND' ) THEN
           DO ic = 1, 20
```

```
freqHRS(ic) = freqND(ic)
           enddo
        ELSE IF ( satelite .EQ. 'NE' ) THEN
           DO ic = 1, 20
              freqHRS(ic) = freqNE(ic)
        ELSE IF ( satelite .EQ. 'NF' ) THEN
           DO ic = 1, 20
              freqHRS(ic) = freqNF(ic)
        ELSE IF ( satelite .EQ. 'NG' ) THEN
           DO ic = 1, 20
              freqHRS(ic) = freqNG(ic)
        ELSE IF ( satelite .EQ. 'NH' ) THEN
           DO ic = 1, 20
              freqHRS(ic) = freqNH(ic)
           enddo
        ELSE
           PRINT *, 'CHACON: UNKNOWN SATELLITE'
           STOP 12
        END IF
   FILL IN VALUES FOR INFRARED CHANNELS , slots 1-20
     DO ic = 1.20
        freq(ic)= freqHRS(ic)
        IRCHAN(ic) = .true.
        MWCHAN(ic) = .false.
      enddo
     TPLCON1 = PLCON1
TPLCON2 = PLCON2
     do ic = 1, 20
        planck2(ic) = freq(ic)
        planck1(ic) = planck2(ic) * planck2(ic)
        planck1(ic) = tplcon1 * planck2(ic) * planck1(ic)
        planck2(ic) = tplcon2 * planck2(ic)
        abscof(ic) = abscof_hrs(ic)
        degchn(ic) = degchn_hrs(ic)
        contmp(ic) = contemp_hrs(ic)
        chscatt(ic) = chscat_hrs(ic)
        em_land(ic) = em_land_hrs(ic)
        em_water(ic) = em_water_hrs(ic)
        dfxcon(ic) = dfx_hrs(ic)
        use_refl(ic) = .false.
      enddo
c "optically thin" HIRS channels which use integrated radiance (reflected
```

from surface) in downward flux calculation.

```
use_refl(8) = .true.
     use_refl(10) = .true.
     use_refl(18) = .true.
     use_refl(19) = .true.
I-----
   FILL IN VALUES FOR MICROWAVE CHANNELS , slots 21-24
     DO ic = 1, 4
       freq(ic+20) = freqMSU(ic)
       IRCHAN(ic+20) = .false.
MWCHAN(ic+20) = .true.
     enddo
! GLA rets had remark about 'old gridding program' and filled in
! different constants for PLCON1 and PLCON2; this has the same
! constants as for IR, but they are not used anyway.
     TPLCON1
               = PLCON1
     TPLCON2
              = PLCON2
     do ic = 21, 24
       planck2(ic) = freq(ic)
       planck1(ic) = planck2(ic) * planck2(ic)
       planck1(ic) = tplcon1 * planck2(ic) * planck1(ic)
       planck2(ic) = tplcon2 * planck2(ic)
       abscof(ic) = abscof_msu(ic-20)
       degchn(ic) = degchn_msu(ic-20)
       contmp(ic) = contemp_msu(ic-20)
       chscatt(ic) = chscat_msu(ic-20)
       em_land(ic) = em_land_msu(ic-20)
       em_water(ic) = em_water_msu(ic-20)
       dfxcon(ic) = dfx_msu(ic-20)
       use_refl(ic) = .false.
     enddo
100 continue
    return
!-----
     end
```

dattrf.f - Set up transmittance coefficients

```
!!ROUTINE: dattrf
!!DESCRIPTION: Read rapid algorithm transmittance function coefficients
            Fills common block /taucoef/
            Based on GLA TOVS retrieval code (dattrf and
              related subroutines)
! SYSTEM ROUTINES USED: dexp, dlog, cos
! SUBROUTINES CALLED: none
! INPUT/OUTPUT FILES USED
    TRCOEF ITRFDAT RAPID ALGORITH TRANS. FUN. DATA SET
!!INPUT PARAMETERS:
! itrfdat - integer unit number for input file
!!OUTPUT PARAMETERS:

    integer error flag

                            0 = successful
                            1 = error reading dataset
                            2 = fewer than 24 channels found
!!REVISION HISTORY:
! 22sep95 Meta S. original routine derived from subroutine dattrf
                in GLA retrieval code
! 16apr96 Meta S. slight modification, clarification, add error flag
 13jul96 Meta S. changed do-loops to remove statement numbers
! REMARKS:
    some changes will need to be made to accomodate multiple
    satellites
subroutine dattrf(itrfdat, ierr)
    implicit none
    integer itrfdat
    integer ierr
include "tovsparam.h"
    include "chacons.h"
    include "matcons.h"
    include "phycons.h"
```

```
include "taucoef.h"
! coefficients for rapid transmittance algorithm
! zao, zbo, zco, zdo, zfo : "old" values read from file from OLDLEV level fit
! za, zb, zc, zd, zf : transformed to maxlev level
! ze, zg
                     : other coefficients
integer oldlev
    parameter (oldlev = 66)
    CHARACTER*8
                 ZCHANID
    DIMENSION ZA
                  ( maxlev , maxangs )
    DIMENSION zZA ( maxlev , maxangs )
    DIMENSION ZB ( maxlev , maxangs )
    DIMENSION ZC ( maxlev , maxangs )
DIMENSION ZC ( maxlev , maxangs )
DIMENSION ZE ( maxlev , maxangs )
DIMENSION ZE ( maxangs )
DIMENSION ZF ( maxlev , maxangs )
DIMENSION ZG ( maxangs )
DIMENSION ZG ( maxangs )
    DIMENSION ZBO ( OLDLEV , maxangs )
    DIMENSION ZCO ( OLDLEV , maxangs )
    DIMENSION ZDO ( OLDLEV , maxangs )
DIMENSION ZFO ( OLDLEV , maxangs )
    DOUBLE PRECISION ZA, ZB, ZC, ZD, ZE, ZM, ZF, ZG, ZN, ZQ,
                    ZAO, ZBO, ZCO, ZDO, ZFO,zza, zzm, zzn
     integer ii,jj,n,l,k
     integer nfound ! counter for no. of channels
     integer yang ! number of coeff. input angles
     integer ylev ! number of coeff. input levels
     ierr = 0
set up angles used in rapid algorithm
NUMANGS
               = 3
     ANGLE(1) = 0.0
    ANGLE(2) = 50.0
    ANGLE(3) = 75.0
    DO II = 1, NUMANGS
       ANGLE(II) = ANGLE(II) * DEG2RAD
       SECANG(II) = COS( ANGLE(II) )
       SECANG(II) = 1.0 / SECANG(II)
     enddo
```

```
READ DATA FOR NEXT CHANNEL FROM TRCOEF
         ______
     REWIND
                ITRFDAT
     nfound
                = 0
 100 CONTINUE
        READ (ITRFDAT, END=600, ERR=500) ZCHANID, N, YANG, N, YLEV,
                    ZM, (ZE(N), N = 1, YANG),
    $
                    ZN, (ZG(N), N = 1, YANG),
    $
                ((ZAO(L,N), ZBO(L,N), ZCO(L,N), ZDO(L,N), ZFO(L,N),
    $
                                     L = 1, YLEV ), N = 1, YANG )
        if ((YANG .NE. 3) .or.
             (YLEV .NE. OLDLEV ) ) go to 100
    $
 we have proper number of coefficients; convert from old 66 levels
  to new 71 levels
I-----
      SHIFT OLDLEV(66) TO NEW MAXLEV(71) FOR ZA ZB ZC ZD ZF
      coefficients zao, zbo, zco, zdo, zfo are at old 66 levels
! the formula \ln ZA(1) = .1 \ln ZAO(1) for 0.1 MB
! the formula \ln ZA(ii) = .3 \ln ZAO(1) for 0.4, 0.7, 1.0 MB
! the formula \ln ZA(5) = .3 \ln ZAO(2) for 1.3 MB
! the formula \ln ZA(6) = .4 \ln ZAO(2) for 1.7 MB
! the formula \ln ZA(7) = .3 \ln ZAO(2) for 2.0 MB
! set ZB, ZC, ZD, ZF = 0.0 from 0.1 MB to 2.0 MB
! make this change for new MAXLEV jack maa 4/27/92
     do jj = 1, yang
       if (ZAO(1,jj) .ne. 0.0) then
     ZA(1,jj) = DEXP(0.1 * DLOG(ZAO(1,jj)))
     do ii=2,4
        ZA(ii,jj) = DEXP(0.3 * DLOG(ZAO(1,jj)))
           end do
       else
     do ii=1,4
        ZA(ii,jj) = 0.0
          end do
       end if
       if (ZAO(2,jj) .ne. 0.0) then
     ZA(5,jj) = DEXP(0.3 * DLOG(ZAO(2,jj)))
     ZA(6,jj) = DEXP(0.4 * DLOG(ZAO(2,jj)))
     ZA(7,jj) = DEXP(0.3 * DLOG(ZAO(2,jj)))
       else
     do ii=5,7
        ZA(ii,jj) = 0.0
          end do
       end if
       do ii=1,7
    ZB(ii,jj) = 0.0
```

```
ZC(ii,jj) = 0.0
    ZD(ii,jj) = 0.0
    ZF(ii,jj) = 0.0
       end do
       do ii=1, OLDLEV - 2
    ZA(ii+7,jj) = ZAO(ii+2,jj)
    ZB(ii+7,jj) = ZBO(ii+2,jj)
    ZC(ii+7,jj) = ZCO(ii+2,jj)
    ZD(ii+7,jj) = ZDO(ii+2,jj)
    ZF(ii+7,jj) = ZFO(ii+2,jj)
       end do
     end do
! look for matching 'chanid' to find
! which slot to copy the coefficients into
       DO K = 1, maxcha
          IF ( ZCHANID .EQ. CHANID(K)) THEN
            nfound = nfound + 1
!-----
     CHANGE UNITS AND TRANSFER COEFFICIENTS TO COMMON /TRCOEF/
   zZM, zZN conversion factors from g/cm^2 to molecules/cm^2
   raised to the power M or N as appropriate
             TAUCFM(K) = ZM
             IF ( ZM .LT. 0.99 ) THEN
               zZM = (AVOGAD / H2OMOL)**ZM
             ELSE
                         = ( AVOGAD / H2OMOL )
               zZM
             ENDIF
             TAUCFN(K) = ZN
             IF ( ZN .LT. 0.99 ) THEN
                     = ( AVOGAD / OZOMOL )**ZN
               zZN
             ELSE
                      = ( AVOGAD / OZOMOL )
               zZN
             ENDIF
! in code portion below...
       zZA -> TAUCFW calculation of d tau^o/dp (not fct. of temperature)
                     part of equation (A6) in Susskind et al 1983
       the 1/(1-tau^o) is included in TAUCFC (i.e. Cij of eqn A7)
             DO N = 1, NUMANGS
                TAUCFE(N,K) = ZE(N)
                TAUCFG(N,K) = ZG(N)
                         = 0.0
               DO L = 1, maxlev
                  TAUCFA(L,N,K) = ZA(L,N)
```

```
TAUCFB(L,N,K) = ZB(L,N)
              zZA(L,N) = (1.0 - ZQ) * (1.0 - ZA(L,N))
              TAUCFW(L,N,K) = zZA(L,N)
              ZQ = ZQ + zZA(L,N)
              IF ( ZQ .GT. 0.0001 ) THEN
                TAUCFC(L,N,K) = ZC(L,N) / ZQ
              ELSE
                TAUCFC(L,N,K) = 0.0
              ENDIF
              TAUCFD(L,N,K) = ZD(L,N) * zZM
              TAUCFF(L,N,K) = ZF(L,N) * zZN
            ENDDO
          ENDDO
! channel slot was found, go back and read another record
          GOTO 100
        ENDIF
      enddo
!-----
        GO BACK TO STMT 100 AND READ ANOTHER RECORD
!-----
   GOTO
            100
!-----
     READ ERROR OR UNEXPECTED END-FILE ON TRFDAT
 500 print *, 'error reading trfdat'
          EOF ON TRFDAT TRYING TO READ NEXT HEADER
!-----
 600 CONTINUE
    IF ( nfound .LT. 24 ) then
      print *, 'error reading dataset'
      ierr = 2
    endif
    close ( itrfdat )
   RETURN
   END
```

grdcon.f – Set vertical grid constants

```
!!GRDCON
!!ROUTINE: grdcon
             SETS VERTICAL GRID CONSTANTS for 71 level
!!DESCRIPTION:
             GLA rapid transmittance algorithm
             initializes common block /GRDCONS/
              Adapted from GLA TOVS retrieval code
              (subroutines grdcon and retini)
! SYSTEM ROUTINES USED: alog
! SUBROUTINES CALLED: none
!!INPUT PARAMETERS: none
!!OUTPUT PARAMETERS: none - sets common block GRDCONS
!!REVISION HISTORY:
                 original routine adapted from GLA TOVS grdcon and retini
! 22Sep95 Meta S.
! 18May96 Meta S.
                 Add calculation for delprs - moved from tovsh2o_tau
subroutine grdcon
     implicit none
     include "tovsparam.h"
     include "phycons.h"
     include "grdcons.h"
     integer 1
! 71 pressure levels
    data
                    pres
    $
           / 0.1,
                   0.4, 0.7, 1.0, 1.3, 1.7,
    $
                              2.,
                                  3.,
                                       4.,
                       7.,
    $
                   6.,
                              8.,
                                   9., 10.,
              5.,
    $
                   20., 30., 40., 50.,
              15.,
    $
              70., 80., 90., 100., 110., 120.,
    $
             130., 140., 150., 160., 170., 180.,
    $
             190., 200., 220., 240., 260., 280.,
    $
             300., 320., 340., 360., 380., 400.,
    $
             425., 450., 475., 500., 525., 550.,
    $
             575., 600., 625., 650., 675., 700.,
    $
             725., 750., 775., 800., 825., 850.,
    $
             875., 900., 925., 950., 975.,1000.,
            1025.,1050.
```

```
! reference temperature for 71 levels
     data
                       stntmp
    $
             /221.6, 254.8, 264.4, 266.5, 265.0, 261.8,
    $
                                   259.7, 251.4, 246.1,
              242.2, 239.2, 236.6, 234.4, 232.5, 230.8,
              228.0, 224.5, 221.0, 217.0, 213.5, 210.6,
              208.1, 206.8, 206.5, 206.1, 206.6, 207.9,
    $
              209.0, 210.1, 211.0, 212.2, 213.4, 214.7,
              215.8, 216.9, 219.3, 222.9, 226.2, 229.3,
    $
              232.1, 235.2, 238.5, 241.6, 244.5, 247.3,
              250.2, 253.3, 256.3, 259.0, 261.6, 263.9,
    $
              266.1, 268.2, 270.3, 272.2, 274.1, 275.9,
              277.5, 278.9, 280.3, 281.6, 282.9, 284.2,
              285.3, 286.3, 287.2, 288.2, 289.1, 290.0,
    $
              290.8, 291.7
! aerosol scattering factor for 71 levels
     data
    $
             /0.00000, 0.00000, 0.00000, 0.00000, 0.00000,
                                         0.00000, 0.00000, 0.00000,
              0.00000, 0.00000, 0.00000, 0.00000, 0.00000,
              0.00000, 0.00000, 0.00000, 0.00000, 0.00000, 0.00022,
    $
              0.00058, 0.00091, 0.00121, 0.00143, 0.00167, 0.00186,
              0.00207, 0.00222, 0.00238, 0.00254, 0.00271, 0.00292,
              0.00300, 0.00300, 0.00300, 0.00300, 0.00300, 0.00300,
              0.00300, 0.00300, 0.00300, 0.00300, 0.00300,
              0.00300, 0.00300, 0.00300, 0.00306, 0.00312, 0.00318,
              0.00325, 0.00331, 0.00340, 0.00354, 0.00368, 0.00381,
              0.00409, 0.00438, 0.00446, 0.00501, 0.00560, 0.00619,
              0.00678, 0.00741, 0.00864, 0.00988, 0.01112, 0.01236,
              0.01359, 0.01483
  calculate layer average of standard temperature and of rapid alg. pressures
  and layer difference in pressure (times a constant)
  NOTE: gravcon is set in phycon.f which must be called prior to this
          routine
     tstd_av(1)
                  = stntmp(1)
     presa(1)
                  = 0.50*pres(1)
     delprs(1) = pres(1) * gravcon
     do 1 = 2, maxlev
                  = 0.5 * (pres(l-1) + pres(l))
        presa(1)
        tstd_av(1) = 0.5 * (stntmp(1-1) + stntmp(1))
        delprs(1) = (pres(1) - pres(1-1)) * gravcon
      enddo
```

```
!
! calculate logarithm of rapid algorithm pressure level values
!
    do l = 1, maxlev
        presln(l) = alog ( pres (l) )
    enddo
    return
    END
```

hirsrad.f - Calculate radiance

```
!!ROUTINE: hirsrad
!!DESCRIPTION: Calculate radiance from input temperature, specific
              humidity, ozone using input transmittance functions
              Based on GLA TOVS retrieval code (vecrad and
                related subroutines)
! SYSTEM ROUTINES USED: cos
! SUBROUTINES CALLED: planck
!!INPUT PARAMETERS:
! tau - real array (of size maxlev by maxcha) of transmittance
! tausun - real array (of length maxcha) of transmittance along
               solar path to Earth's surface
! nchan - integer number of channels to calculate
! chanarr - integer array (of length maxcha) of channels to calculate
               1-20 -> HIRS2 ch 1-20; 21-24 -> MSU 1-4
! tair - real array (of length maxlev) with temperature profile (K)
! tmptop - real temperature (K) at "top of atmosphere"
! ir_zen - real
                    satellite zenith angle for infrared channels (degrees)
! emissmw - real microwave emissivity
! land
         - logical flag (.true. if land, .false. if water)
! nlev
          - real number of levels to use in profiles
! rho
         - real bidirectional reflectance
! sunzang - real solar zenith angle (degrees)
! dayflag - logical flag for day/night (.true. if day, .false. if night)
! tground - real skin (ground) temperature (K)
! tsurfair - real surface air temperature (K)
!!OUTPUT PARAMETERS:
  calc_rad - real array (of length maxcha) of radiance (or brightness
                temperatures, for MSU channels ) calculated from profile
                for channels input in 'chanarr'
!!REVISION HISTORY:
 08sep95 Meta S. original routine derived from subroutine vecrad
                  in GLA retrieval code
                  make solar calculation (which was borrowed from
 03oct95 Meta S.
                  GLA routine grtemp) more consistent with
                  solar calculations in tows_tauang. Revised prologue.
  15dec95 Meta S.
                  ensure rad_down is initialized
                  *Input arguments changed* so multiple channels can
 17apr96 Meta S.
                  be calculated with a single subroutine call
                  (requested "optimization" change)
```

```
subroutine hirsrad(tau,tausun,nchan,chanarr,tair,tmptop,ir_zen,
           emissmw, land, nlev, rho, sunzang, dayflag, tground,
    &
    &
          tsurfair, calc_rad)
      implicit none
      include "tovsparam.h"
! input variables
     real
                tau(maxlev, maxcha)
                tausun(maxcha)
     real
                nchan
     integer
     integer
                chanarr(maxcha)
                tair(maxlev)
     real
     real
                tmptop
     real
                ir_zen
     real
                emissmw
     logical
                land
     integer
                nlev
     real
                rho
     real
                sunzang
     logical
                dayflag
     real
                tground
     real
                tsurfair
! output variable
     real
                calc_rad(maxcha)
! function types
     real
                planck
! parameters from common
      include "phycons.h"
      include "chacons.h"
      include "matcons.h"
! local variables
     real
                angsun
     real
                               ! blackbody temp. of avg layer temperature
                btav
     real
                               ! blackbody temp. of ground temperature
                btgnd
                              ! blackbody temp. of surface air temperature
     real
                btsurf
                              ! channel counter
     integer
                chan
     real
                crad
                               ! partial sum for radiance
                               ! difference of transmittance in layer
     real
                deltau
     logical
                dosun
     real
                dtauinv
                               ! inverse transmittance difference
                               ! sfc emissivity for this sounding
     real
                emiss
     integer
                              ! counter
                i,l
     real
                pcon1
                pcon2
     real
                              ! atmos. emission downward flux
     real
                rad_down
                              ! solar radiation at TOA (?)
     real
                rad_sun
```

```
rhosec
     real
     real
                               ! solar contribution to calc. radiance
                sunterm
     real
                tav(maxlev) ! mean layer temperature array
     tav(1) = 0.5 * (tair(1) + tmptop)
     do l = 2, nlev
         tav(1) = 0.5 * (tair(1-1) + tair(1))
      enddo
     do i = 1, nchan
         chan = chanarr(i)
         dosun = ( freq(chan) .gt. 2000. ) .and. dayflag
         pcon1 = planck1(chan)
        pcon2 = planck2(chan)
         if (irchan(chan)) then
! initialize radiance sum
           deltau = 1.0 - tau(1,chan)
           btav = planck(tav(1), pcon1, pcon2)
           crad = btav * deltau
! integrate through atmosphere (from top down)
            do 1 = 2, nlev
              btav = planck(tav(1), pcon1, pcon2)
               deltau = tau(l-1,chan) - tau(l,chan)
              crad = crad + btav * deltau
            enddo
! prepare to calculate surface contribution
           btsurf = planck(tsurfair, pcon1, pcon2)
           btgnd = planck(tground, pcon1, pcon2)
           rad_down = dfxcon(chan) * btsurf * (1. - tau(nlev, chan))
            if (land) then
               emiss = em_land(chan)
            else
               emiss = em_water(chan)
            endif
         else if (mwchan(chan)) then
! initialize radiance sum
           btav = tav(1)
           deltau = 1.0 - tau(1, chan)
           rad_down = 0.0
            if (tau(1,chan) .ne. 0.0) then
              dtauinv = (1. / tau(1, chan)) - 1.
              rad_down = dtauinv * btav
            endif
```

```
crad = btav * deltau
! integrate through atmosphere (from top down)
            do l = 2, nlev
               btav = tav(1)
               deltau = tau(l-1, chan) - tau(l, chan)
               if ((tau(1,chan) .ne. 0.0) .and.
                    (tau(1-1,chan) .ne. 0.0) ) then
    &
                  dtauinv = (1. / tau(l,chan)) - (1. / tau(l-1,chan))
                  rad_down = rad_down + dtauinv * btav
               endif
               crad = crad + btav * deltau
            enddo
! prepare to calculate surface contribution
           btsurf = tsurfair
           btgnd = tground
           rad_down = (rad_down + bigbang) * tau(nlev,chan)
            emiss = emissmw
         endif
         if (use_refl(chan)) then
            crad = crad + tau(nlev, chan) *
    &
              (cos(ir_zen*deg2rad)*crad*(1.-emiss)*(dfxcon(chan)*2.0)
                 + emiss * btgnd)
    &
         else
           crad = crad + tau(nlev, chan) *
                 (rad_down + emiss * ( btgnd - rad_down ))
    &
         endif
! Add solar contribution for wavenumber greater than 2000.0
         if (dosun) then
            if (sunzang .le. sunset) then
               angsun = sunzang * deg2rad
            else
               angsun = 89.9e0 * deg2rad
            endif
           rhosec = rho * cos(angsun)
            rad_sun = scon * planck(tsun, pcon1, pcon2)
            sunterm = rhosec * rad_sun * tausun(chan)
           crad = crad + sunterm
         endif
         calc_rad(chan) = crad
      enddo
      return
      end
```

initialGLA.f - Sample initialization code

```
!!ROUTINE: initialGLA
!!DESCRIPTION: *Sample* routine to carry out initialization
              of variables ... derived from code fragments in
              test routines
! SYSTEM ROUTINES USED:
! SUBROUTINES CALLED:
   matcon - SETS MATH CONSTANTS FOR RETRIEVALS
   phycon - SETS PHYSICAL CONSTANTS FOR RETRIEVALS
   grdcon - SETS VERTICAL GRID CONSTANTS
   chacon - Set channel constants for TOVS (HIRS/MSU) channels
   dattrf - Read rapid algorithm transmittance function coefficients
! FILES NEEDED: "bland" land/water flags
!!INPUT PARAMETERS:
! itrfdat - integer unit number for input file
! satelite - character*2 identifier for satellite
!!OUTPUT PARAMETERS:
         - logical array (of size 360 by 180) of 1X1 degree flags
            for land and water
         - integer error flag for transmittance coeffs
 ierr
                            0 - successful read
                            1 - error reading dataset
                            2 - less than 24 channels found
!!REVISION HISTORY:
! 29nov95 Meta S. original routine
! 28mar96 Meta S. added prologue
! 19may96 Meta S.
                 shifted order of subroutine calls: matcon before
                   phycon (before grdcon)
subroutine initialGLA(itrfdat, satelite, bland, ierr)
ŗ
    character*2 satelite
     integer itrfdat
    logical bland(360,180)
     integer ierr
```

```
integer i,j
! initialize constants, physical, mathematical, and vertical grid related
     call matcon
     call phycon
     call grdcon
!
     read in land/water flag dataset
! the following statement will need to be changed for the operational
! system to access a more general input dataset
     open(31,file='/ford1/local/new-data-types/gla-tovs/data/bland',
           form='formatted',status='old')
     do i = 1,360
       read(31,'(18011)') (bland(i,j),j=1,180)
     enddo
! initialize channel-related constants, read in transmittance coefficients
     call chacon(satelite)
     call dattrf(itrfdat,ierr)
     return
     end
```

matcon.f - Set up mathematical constants

```
!!ROUTINE: MATCON
!!DESCRIPTION: SETS MATH CONSTANTS FOR RETRIEVALS
! SYSTEM ROUTINES USED: none
! SUBROUTINES CALLED: none
!!INPUT PARAMETERS: none
!!OUTPUT PARAMETERS: none
!!REVISION HISTORY:
         1nov84 Jim Pf. original routine in GLA retrival code
        15sep95 Meta S. change common /matcons/ to include
                  only trigonometric constants
SUBROUTINE MATCON
   implicit none
   include "matcons.h"
l_____
   SET MATHEMATICAL CONSTANTS AS USED IN GLA RETS
           = 3.141592654
   DEG2RAD
          = PI / 180.0
   PID2
           = PI / 2.0
          = 180.0 / PI
   RAD2DEG
           = PI + PI
   TWOPI
   RETURN
   END
```

phycon.f - Set up physical constants

```
!!ROUTINE: PHYCON
!!DESCRIPTION:
            SETS PHYSICAL CONSTANTS FOR RETRIEVALS
             fills GLA common 'phycons'
! SYSTEM ROUTINES USED:
                    NONE
! SUBROUTINES CALLED:
                    NONE
!!INPUT PARAMETERS: none
!!OUTPUT PARAMETERS: none, fills GLA common 'phycons'
!!REVISION HISTORY:
          2Apr85 Jim Pf. original routien in GLA retrieval code
         15sep95 Meta S. reformatted original GLA routine
         28mar95 Meta S. removed some unsed constants,
                          added some comments
         18may95 Meta S. move some "constants" from tovsh2o_tau
subroutine phycon
    implicit none
    include "matcons.h"
    include "phycons.h"
    local variables
    real radsun
                 ! Sun's radius
    real radeorb
                 ! radius of Earth's orbit around Sun
._____
  set physical constants as used in GLA retrievals
!-----
              = 6.022045e+23
                                ! molecules/mole
    avogad
    boltzmns = 8.317e0 / avogad ! Joule / Kelvin clight = 2.9979e+10 ! cm s^{-1} declmax = 23.45 ! degrees plancks = 6.626176e-34 ! Joule-sec radeorb = 149.57e+09 ! meters radsun = 0.69595e+09 ! meters
    atmosph
            = 1013.25
                                ! millibars
    bigbang
            = 2.7
                                ! Kelvin
            = 9.80665
                               ! m s^{-2}
    grav
    daymaxd
              = 173.0
```

planck.f - Calculate Planck function

```
!!ROUTINE: PLANCK
!!DESCRIPTION: COMPUTES RADIANCE FROM TEMPERATURE VIA PLANCK FUNCTION
! SYSTEM ROUTINES USED: none
! SUBROUTINES CALLED: none
!!INPUT PARAMETERS:
            real Temperature (K)
    temp
            real WAVE NUMBER DEPENDENT CONSTANT
    PLANK1
                  PLANK1 = 2 * VELLIGHT**2 * PLANCKCON / LAMDA**3
                  PLANK1 = 1.193E-12/LAMDA**3 WATTS / (CM**2 - SR)
                    WHERE
                  VELLIGHT = 3.00E+10 CM/SEC
                  PLANKCON = 6.63E-34 JOULES-SEC
                         = WAVE LENGTH IN CM
                 LAMDA
    PLANK2
            real WAVE NUMBER DEPENDENT CONSTANT
                  PLANK2 = PLANCKCON*VELLIGHT / (BOLTZMAN*LAMDA)
                  PLANK2 = 1.441/LAMDA
                                    DEG.K
                    WHERE
                  BOLTZMAN = 1.38E-23 JOULE / DEG.K
!!OUTPUT PARAMETERS:
    PLANCK real blackbody radiance for given temperature
               UNITS WATTS/(CM**2-SR)
!!REVISION HISTORY:
    25oct1984 Jim Pf.
                      original routine VPLANCK in GLA retrieval code
    11oct1995 Meta S.
                      added DAO prologue
real FUNCTION PLANCK(temp,PLANCK1,PLANCK2)
    implicit none
! STATEMENT FUNCTION FOR PLANCK FUNCTION
    real
           temp
    real
           planck1
    real
           planck2
    PLANCK = PLANCK1/(EXP(PLANCK2/temp)-1.0)
    RETURN
    END
```

tovsfix_tau.f - Fixed gas transmittance contribution

```
!!TOVSFIX TAU
!!ROUTINE: tovsfix_tau
!!DESCRIPTION: Calculate contribution to transmittance by fixed gases
             using rapid transmittance algorithm coefficients
             - reference Susskind, et al. 1983 JGR, p 8565
! CALLED FROM:
             tovs_tau
! SYSTEM ROUTINES USED: none
! SUBROUTINES CALLED: none
!!INPUT PARAMETERS:
        - real array (of length nlev) with difference between
             current best estimate of layer mean temperatures and
             standard atmosphere layer mean temperatures (K)
! chan
         - integer channel number
! nlev
         - integer surface level (also number of levels to use)
!!OUTPUT PARAMETERS:
! taufix - real array of effective mean layer transmittance
            for fixed gases
!!REVISION HISTORY:
 07sep95 Meta S. original routine derived from subroutine tmptau
                 in GLA retrieval code
 18may96 Meta S. effTdif need not be saved as fct. of angle
subroutine tovsfix_tau(tdif, chan, nlev, taufix)
     implicit none
     include "tovsparam.h"
 variables passed into routine
     integer nlev, chan
    real tdif(maxlev)
! variables output from routine
    real taufix(maxlev,maxangs)
! constants, quasi-constants, and coefficients
     include "taucoef.h"
```

```
! local variables
     integer 1,k
     real effTdif
 Susskind et al 1983 algorithm for transmittance from fixed gases
  Effective mean temperature above P -> \tilde{T} in equation (A6)
! taucfw: precomputed non-varying part of \tilde{T} integral
  taucfc: includes term with division by (1 - \tau^o)
    here effTdif -> \tilde{T} - \tilde{T}^o or
į
      difference between effective mean temperatures for the
      current temperature profile and the standard temperature profile
  Then:
  tau = A + B (T - T^o) + C ( \tilde{T} - \tilde{T}^o)
                                                          (A7)
     do k = 1, maxangs
        effTdif = 0.
        do l = 1, nlev
           effTdif = effTdif + taucfw(1,k,chan) * tdif(1)
           taufix(1,k) = taucfa(1,k,chan) + taucfb(1,k,chan) * tdif(1)
                + taucfc(1,k,chan) * effTdif
    &
           taufix(l,k) = amax1(taufix(l,k), 0.0)
           taufix(l,k) = amin1(taufix(l,k), 1.0)
        enddo
      enddo
     return
      end
```

tovsh2o_tau.f - Water vapor transmittance contribution

```
!!TOVSH2O TAU
!!ROUTINE: tovsh2o_tau
!!DESCRIPTION: Calculate contribution to transmittance by water vapor
             using rapid transmittance algorithm coefficients
             - reference Susskind, et al. 1983 JGR, p 8564
! CALLED FROM:
             tovs_tau
! SYSTEM ROUTINES USED: exp
! SUBROUTINES CALLED: none
!!INPUT PARAMETERS:
! h2ocd - real array (of length maxlev) with water vapor layer
              column density ( g cm^-2)
         - real array (of length maxlev) of H2O vapor pressure (atm)
! chan
         - integer channel number
         - integer surface level (also number of levels to use)
! tav - real array (of length maxlev) with temperature profile (K)
! celsius - real array (of length maxlev) with (T - 273) (approx.
               temperature in celsius)
!!OUTPUT PARAMETERS:
! tauh2o - real array of effective h2o transmittance
!!REVISION HISTORY:
! O6sep95 Meta S. original routine derived from subroutine h2otau
                 in GLA retrieval code
 17may96 Meta S.
                 moved pressure calculations to main routine,
                 rewrote continuum expression
subroutine tovsh2o_tau( h2ocd, e, chan, nlev, tav,
              celsius, tauh2o)
    &
     implicit none
     include "tovsparam.h"
! variables passed into routine
    real
           h2ocd(maxlev)
    real
            e(maxlev)
    integer chan
    integer nlev
    real
           tav(maxlev)
    real
          celsius(maxlev)
```

```
! variable output from routine
      real
               tauh2o(maxlev,maxangs)
! constants, quasi-constants, and coefficients
      include "taucoef.h"
      include "chacons.h"
      include "grdcons.h"
! local variables
             cont(maxlev)
      real
              effh2o(maxlev)
      real
      integer 1, k
      real
               taulog
! statement function definitions
      real
               quikexp
      real
      quikexp(X) = 1.0 + X * (1.0 + X * (.5 + X * .16666667))
! for IR channel, calculate continuum (except for angle contribution)
     see Susskind and Searl (1978) J. Quant. Spect. Radiat. Transfer
! \quad kH2O(\nu,i) = \sum_{k=1}^{\infty} k2(\nu,T) e(1) U(1)
! U(1) (as above) --> h2ocd = delprs * h2omid
! degchn = (-dK/dT) for channel; k2(T) = k2(contmp)*(1-degchn)(T-contmp)
į
      if (irchan(chan)) then
         do l = 1, nlev
            cont(1) = e(1) * abscof(chan) * h2ocd(1) *
                 (1.0 - degchn(chan) * (tav(1) - contmp(chan)))
     &
         enddo
      else
         do l = 1, nlev
            cont(1) = 0.0
         enddo
      endif
      if (taucfm(chan).lt. 0.99) then
         do 1 = 1,nlev
            effh2o(1) = h2ocd(1) ** taucfm(chan)
         enddo
      else
         do l = 1, nlev
            effh2o(1) = h2ocd(1)
```

```
endif
! From (A5) in Susskind, et al. 1983 (page 8564)
! tau(H2O) = exp(-(D(1 - E(T - 273)) h2ocd**M)
         { * exp(-sec \theta * KcontinuumH2O) of course }
  see also ON-9608, eq32
! From GLA retrieval code:
    TAULOG(II,K) = (-1.0 - TAUCFE(K,YCHA) * TMPXCES(II))
                   * TAUCFD(LEVL,K,YCHA) * EFFH2O(II)
    so perhaps TAUCFE = -E
     do k = 1, numangs
        do l = 1, nlev
           taulog = (-1.0 - taucfe(k, chan) * celsius(l))
    &
                * taucfd(1,k,chan) * effh2o(1)
           taulog = taulog - secang(k) * cont(l)
           taulog = amin1(taulog, 0.0)
           tauh2o(1,k) = quikexp( taulog )
        enddo
     enddo
```

enddo

return end

tovsozo_tau.f - Ozone transmittance contribution

```
!!TOVSOZO TAU
!!ROUTINE: tovsozo_tau
!!DESCRIPTION: Calculate contribution to transmittance by ozone
             using rapid transmittance algorithm coefficients
             - reference Susskind, et al. 1983 JGR, p 8564
! CALLED FROM:
             tovs_tau
! SYSTEM ROUTINES USED: exp
! SUBROUTINES CALLED: none
!!INPUT PARAMETERS:
! ozomid - real array (of length mlev) with ozone column
                densities (g/cm**2)
         - integer channel number
! chan
         - integer number of levels (top to surface)
! nlev
 celsius - real array (of length mlev) with profile of (T-273) (K)
!!OUTPUT PARAMETERS:
        - real array (of size maxlev by maxangs) of effective ozone
  tauozo
           transmittance
!!REVISION HISTORY:
 06sep95 Meta S. original routine derived from subroutine ozotau
                 in GLA retrieval code
subroutine tovsozo_tau(ozomid, chan, nlev, celsius, tauozo)
    implicit none
    include "tovsparam.h"
! variables passed into routine
    real
          ozomid(maxlev)
    integer chan
    integer nlev
            celsius(maxlev)
    real
! variable output from routine
           tauozo(maxlev, maxangs)
    real
! constants, quasi-constants, and coefficients
    include "taucoef.h"
```

```
! local storage
              eff03(maxlev)
     real
      integer 1, k
     real
              taulog
! statement function definitions
     real
              quikexp
     real
              χ
     quikexp(X) = 1.0 + X * (1.0 + X * (.5 + X * .16666667))
  calculate 'eff03' as (ozone conc.) ** taucfN
  following is formula used in GLA code:
       if (taucfn(chan) .1t. 0.99) then
         do l = 1, nlev
            eff03(1) = taucfn(chan) * alog( ozomid(1))
            eff03(1) = exp(eff03)
          enddo
      else
         do l = 1, nlev
            eff03(1) = ozomid(1)
          enddo
     endif
     do l = 1, nlev
         eff03(1) = ozomid(1) ** taucfn(chan)
      enddo
! From (A5) in Susskind, et al. 1983 (page 8564)
! tau(ozone) = exp( - (F(1 - G(T - 273)) ozomid**N)
                                              see also ON-9608 eq 33
! from GLA retrieval code:
! TAULOG(II,K) = (-1.0 - TAUCFG(K,YCHA) * TMPXCES(II))
   * TAUCFF(LEVL,K,YCHA) * EFFO3(II) * TAUCFF(LEVL,K,YCHA) * EFFO3(II)
! so sign of TAUCFG = -G , perhaps...
     do k = 1, numangs
         do l = 1, nlev
           taulog = ( -1.0 - taucfg(k,chan) * celsius(l))
    &
                 * taucff(1,k,chan) * eff03(1)
           taulog = amin1 (taulog, 0.0)
           tauozo(l,k) = quikexp( taulog )
         enddo
     enddo
     return
      end
```

tovs_tau.f - Driver for transmittance calculation

```
!!ROUTINE: tovs_tau
!!DESCRIPTION: Calculate transmittance for given channel numbers
              for a specified temperature, moisture, and ozone profile
              Based on GLA TOVS retrieval code (comtau and
                 related subroutines)
! SYSTEM ROUTINES USED: abs, cos
! SUBROUTINES CALLED:
   tovsfix_tau - transmittance contribution due to fixed gases
   tovsh2o_tau - transmittance contribution due to H2O
   tovsozo_tau - transmittance contribution due to ozone
   tovs_tauang - interpolate layer transmittance to satellite zenith
                   angle and take products over layers
!!INPUT PARAMETERS:
         - real array (of length maxlev) with temperature profile (K)
! tair
  tmptop
          - real temperature (K) at "top of atmosphere"
  h2omid - real array (of length maxlev) with water vapor layer specific
              humidities (KG/KG)
 ozomid - real array (of length maxlev) with ozone column densities
               (g/cm**2)
          - real surface pressure (mb)
 ps
! nchan - integer number of channels to calculate
  chanarr - integer array (of length maxcha) of channels to calculate
               1-20 -> HIRS2 ch 1-20; 21-24 -> MSU 1-4
! nlev
          - integer surface level (also number of levels to use)
! mw_zen - real
                    satellite zenith angle for microwave channels
 ir_zen
          - real
                    satellite zenith angle for infrared channels
! dayflag - logical true if day ( sun angle < SUNSET = 85.0E0)
  secsun - real 1. / cos( sun angle)
!!OUTPUT PARAMETERS:
         - real array (of size maxlev by maxcha) of transmittance along
             path to satellite.
  tausun - real array (of size maxcha) of transmittance along path from
  errflag - logical returns .true. if error
!!REVISION HISTORY:
! 11sep95 Meta S. original routine derived from subroutine comtau
                  in GLA retrieval code
! 04oct95 Meta S. clarify arrangement of channels in tau, tausun
```

```
! 04dec95 Meta S. update prologue, add comments describing local
                     variables
  17may96 Meta S. moved calculation of column H2O density and vapor
                     pressure from tovsh2o_tau into this routine
subroutine tovs_tau(tair, tmptop, h2omid, ozomid, ps, nchan,
          chanarr, nlev, mw_zen, ir_zen, dayflag, secsun, tau, tausun,
    &
          errflag)
     implicit none
     include "tovsparam.h"
 variables passed into routine
            tair(maxlev)
     real
     real
            tmptop
     real h2omid(maxlev)
real ozomid(maxlev)
     real
            ps
     integer nchan
     integer chanarr(maxcha)
     integer nlev
     real mw_zen
     real
            ir_zen
     logical dayflag
     real
             secsun
! variables output from routine
     real tau(maxlev, maxcha)
     real
             tausun(maxcha)
     logical errflag
! constants, quasi-constants, and coefficients
     include "matcons.h"
     include "grdcons.h"
     include "chacons.h"
     include "phycons.h"
! local variables
     integer 1,k
                                 ! counters
     integer ichan
     integer chan
     real tav(maxlev)
                                 ! mean layer temperature
     real tdif(maxlev)
                                 ! difference from rapid alg std. temp.
     real celsius(maxlev)
                                 ! difference from 273
     real effangmw
                                 ! microwave and
     real seceffmw
     real effangir
                                 ! IR channel zenith angles and secants
```

```
real seceffir
     real effang
     real seceff
     real h2ocd (maxlev) ! H2O column density
     real e(maxlev)
                                   ! H2O vapor pressure
! effective mean layer transmittance from fixed gas contribution
     real taufix(maxlev,maxangs)
! effective mean layer transmittance from water vaper contribution
     real tauh2o(maxlev,maxangs)
! effective mean layer transmittance from ozone contribution
     real tauozo(maxlev,maxangs)
! effective mean layer transmittances at rapid algorithm angles
     real tauang(maxlev,maxangs)
! calculated transmittance along path for satellite zenith angle
! and sun angle
     real ctau(maxlev), ctausun
      errflag = .false.
! calculate quantities which are common for all channels
! tay - mean layer temperature for RT calculation
! tdif - difference from rapid algorithm's std. atmos. mean temp.
! celsius - difference from 273 K
     (note this is not Celsius; 273 K is used in rapid algorithm)
     tav(1) = 0.5 * (tmptop + tair(1))
      tdif(1) = tav(1) - tstd_av(1)
      celsius(1) = tav(1) - 273.
     do l = 2, nlev
         tav(1) = (tair(1-1)+tair(1)) * 0.5
         tdif(1) = tav(1) - tstd_av(1)
         celsius(1) = tav(1) - 273.
      enddo
! compute mid-level h2o column densities (g/cm**2)
   delprs = {\delta p } * 10 / grav --> density(air) * {\delta z}
   delprs * h2omid(spec. humid) --> column density effh2o U(1)
     do l = 1, nlev-1
       h2omid(1) = amax1(h2omid(1), shmin)
       h2ocd(1) = delprs(1) * h2omid(1)
     h2ocd(nlev) = (ps - pres(nlev-1)) * gravcon * h2omid(nlev)
```

```
! e(1) = H2O partial pressure (in atm)
           = (airmol/h2omol) * P(1) / (1013.25 mb per atm)
                       * h2omid (spec. humidity)
!
     econ = airmol / (h2omol * atmosph) ! set in phycons.h
     do l = 1, nlev-1
        e(1) = presa(1) * econ * h2omid(1)
      e(nlev) = (ps + pres(nlev-1)) * 0.5 * econ * h2omid(nlev)
! zenith angle factors for microwave and IR channels
      effangmw = abs( deg2rad * mw_zen )
      seceffmw = 1. / cos( effangmw )
                                           ! secant of MW zenith angle
      effangir = abs( deg2rad * ir_zen )
      seceffir = 1. / cos( effangir )     ! secant of IR zenith angle
! loop over requested angles
      do ichan = 1,nchan ! begin channel loop
        chan = chanarr(ichan)
         if (chan .gt. maxcha) then
           print *,' tovs_tau: bad channel requested'
            errflag = .true.
           return
         else
! set appropriate zenith angle (effang) and secant for IR or MW channel
            if (irchan(chan)) then
              effang = effangir
              seceff = seceffir
            else if (mwchan(chan)) then
               effang = effangmw
              seceff = seceffmw
            endif
! calculate contribution to transmittance from fixed gases
           call tovsfix_tau(tdif, chan, nlev, taufix)
! calculate contribution to transmittance from water vapor
           call tovsh2o_tau( h2ocd, e, chan, nlev, tav,
               celsius, tauh2o)
! calculate contribution to transmittance from ozone
           call tovsozo_tau( ozomid, chan, nlev, celsius, tauozo)
! mean layer tramsmittance is product of contributions from fixed gases,
    H2O and O3; calculate for angles used in rapid algorithm
           do k = 1,maxangs
              do l = 1, nlev
                  tauang(1,k) = taufix(1,k)*tauh2o(1,k)*tauozo(1,k)
```

```
enddo
           enddo
! interpolate transmittance from rapid algorthim angles to observed
     zenith angle and to solar zenith angle
           call tovs_tauang(tauang, chan, nlev, effang, seceff,
    &
                dayflag, secsun, ctau, ctausun)
  fill arrays with calculated tau and tausun
           do 1 = 1,nlev
              tau(1,chan) = ctau(1)
           enddo
           tausun(chan) = ctausun
         endif
     enddo
                                          ! end of channel loop
     return
     end
```

tovs_tauang.f - Interpolate transmittance to zenith angle

```
!!TOVS TAUANG
!!ROUTINE: tovs_tauang
!!DESCRIPTION: Interpolate computed transmittances to satellite
               zenith angle
! CALLED FROM:
              tovs_tau
! SYSTEM ROUTINES USED: exp, cos, acos
! SUBROUTINES CALLED: none
!!INPUT PARAMETERS:
! tauang - real matrix (of size maxlev by maxangs) of mean layer
             transmittance calculated for different zenith angles
! chan
         - integer channel number
         - integer surface level (also number of levels to use)
! nlev
! effang - real abs(zenith angle) in radians
! seceff - real 1. / cos(effang)
! dayflag - logical flag day = .true. night = .false.
         - real secant of solar angle for this observation
!!OUTPUT PARAMETERS:
         - real array of mean layer transmittance
               for satellite zenith angle
  tausun - real mean layer transmittance
               for solar radiation
!!REVISION HISTORY:
! O8sep95 Meta S. original routine derived from subroutine tauang
                 and tscatt in GLA retrieval code
į
   1dec95 Meta S.
                 set tausun = 0 to cover when sun term not used
                 (avoid uninitialized variable)
subroutine tovs_tauang(tauang, chan, nlev, effang, seceff, dayflag,
              secsun, tau, tausun)
     implicit none
     include "tovsparam.h"
! variables passed into routine
            tauang(maxlev,maxangs)
     real
     integer chan
     integer nlev
     real
            effang
```

```
seceff
     real
     logical dayflag
     real
             secsun
! variables output from routine
            tau(maxlev)
     real
     real
              tausun
! constants, quasi-constants, and coefficients
! (in common block or passed into routine)
     include "taucoef.h"
     include "grdcons.h"
     include "chacons.h"
! local variables
     real
             anglsun
     logical dosun
     real
             efcos
     real
              efsun
     integer k, l, nfang, nfsun, nangm1
             sat_factor
     real
     real
             sfactor
             sun_factor
     real
     real
             tauscat
! statement functions
     real quikexp
     real X
     quikexp(X) = 1.0 + X * (1.0 + X * (.5 + X * .16666667))
! set up some variables
     nangm1 = numangs - 1
     dosun = (freq(chan).gt. 2000.).and.dayflag
                       ! set this in case of .not.irchan or .not.dosun
! find which angles of rapid algorithm bracket the observed zenith angle
! (nfang and nfang+1) and calculate factor (sat_factor) for interpolation
  in sec(zenith angle)
     nfang = 1
     do k = 2, nangm1
        if (effang .gt. angle(k) ) nfang = k
     enddo
     sat_factor = ( seceff - secang(nfang) ) /
                      (secang(nfang+1) - secang(nfang) )
 calculate transmittance as product of transmittance through layers above
   and transmittance in current layer (interpolated to sat. zenith angle)
     tau(1) = amax1(0.0, (tauang(1, nfang))
```

```
+ sat_factor * (tauang(1,nfang+1)-tauang(1,nfang)) ) )
     do l = 2, nlev
        tau(1) = tau(1-1) *
             amax1( 0.0, (tauang(l, nfang)
     &
                + sat_factor * (tauang(l,nfang+1)-tauang(l,nfang))))
    &
      enddo
      if (irchan(chan)) then
! for IR channels, include factor for scattering (from GLA routine tscatt)
         do l = 1, nlev
            sfactor = chscatt(chan) * scat(1)
            tauscat = quikexp( -sfactor * seceff )
            tau(1) = tau(1) * tauscat
         enddo
         if (dosun) then
! if solar contribution needed, calculate effective transmittance for sun
! find which angles of rapid algorithm bracket the effective sun angle
! (nfsun and nfsun+1) and calculate factor (sun_factor) for interpolation
! in sec(sun angle)
            anglsun = seceff + secsun
            efcos = 1. / anglsun
            efsun = acos( efcos )
           nfsun = 1
            do k = 2, nangm1
               if ( efsun .gt. angle(k) ) nfsun = k
            enddo
            sun_factor = ( anglsun - secang(nfsun) ) /
                         (secang(nfsun+1) - secang(nfsun) )
    &
! calculate product of transmittance in layers along solar path
            tausun = amax1( 0.0, (tauang(1,nfsun)
              + sun_factor * (tauang(1,nfsun+1) - tauang(1,nfsun))))
    &
            do 1 = 2, nlev
               tausun = tausun *
                 amax1(0.0, (tauang(1, nfsun)
    &
    &r.
                 + sun_factor * (tauang(l,nfsun+1)-tauang(l,nfsun))))
            enddo
            sfactor = chscatt(chan) * scat(nlev)
            tauscat = quikexp( -sfactor * anglsun )
            tausun = tausun * tauscat
                                                  !!! check this...
         endif
      endif
      return
      end
```

B Jacobian modules prologues and source code

dplanck.f - Calculate derivative of Planck function

```
!!DPLANCK
!!ROUTINE: dPLANCK
!!DESCRIPTION: derivative of PLANCK FUNCTION wrt. temperature
! CALLED FROM: dTb_dTQU
! SYSTEM ROUTINES USED: none
! SUBROUTINES CALLED: none
!!INPUT PARAMETERS:
            real Temperature (K)
    temp
    PLANK1 real WAVE NUMBER DEPENDENT CONSTANT
                 PLANK1 = 2 * VELLIGHT**2 * PLANCKCON / LAMDA**3
                 PLANK1 = 1.193E-12/LAMDA**3 WATTS / (CM**2 - SR)
                    WHERE
                 VELLIGHT = 3.00E+10 CM/SEC
                 PLANKCON = 6.63E-34 JOULES-SEC
                        = WAVE LENGTH IN CM
                 T. A MD A
    PLANK2 real WAVE NUMBER DEPENDENT CONSTANT
                 PLANK2 = PLANCKCON*VELLIGHT / (BOLTZMAN*LAMDA)
                 PLANK2 = 1.441/LAMDA
                                   DEG.K
                    WHERE
                 BOLTZMAN = 1.38E-23 JOULE / DEG.K
!!OUTPUT PARAMETERS:
    PLANCK real blackbody radiance for given temperature
               UNITS WATTS/(CM**2-SR)
!!REVISION HISTORY:
                     original routine
    27nov1995 Meta S.
real FUNCTION dPLANCK(temp,PLANCK1,PLANCK2)
    implicit none
! STATEMENT FUNCTION FOR PLANCK FUNCTION
    real temp
    real planck1
    real planck2
    real d1
                      ! temporary variable
```

```
d1 = EXP(PLANCK2/temp)
dPLANCK = PLANCK1 / (d1-1.)**2 * d1 * planck2 / (temp*temp)

RETURN
END
```

dTb_dTQU.f - Calculate brightness temperature Jacobian

```
!!DTB dTQU
!!ROUTINE: dTb_dTQU
!!DESCRIPTION: Calculate Jacobian or derivative of brightness
              temperature with respect to input temperature,
              water vapor and ozone profiles
              dTb/dT, dTb/dq and dTb/du
              Based on GLA TOVS retrieval code (vecrad and
                 related subroutines)
! CALLED FROM: tovs_tauK
! SYSTEM ROUTINES USED: cos
! SUBROUTINES CALLED: planck, dplanck
!!INPUT PARAMETERS:
! tau - real array (of size maxlev) of transmittance
! tausun - real transmittance along solar path to Earth's surface
 chan
          - integer channel index for current calculation
! tair
          - real array (of length maxlev) with profile of 71-level
               temperatures (K)
! ir_zen - real
                    satellite zenith angle for infrared channels (degrees)
  emissmw - real microwave emissivity
! land - logical flag (.true. if land, .false. if water)
         - real number of levels to use in profiles
! nlev
! rho - real bidirectional reflectance
! secsun - real secant of solar angle for this observation
          - logical flag .true. if solar calculations needed
! tground - real skin (ground) temperature (K)
! tsurfair - real surface air temperature (K)
          - real array (of size maxpack) of derivatives of
                 tau with respect to changes in temperature at given layers
! packed matrix:
                   first dimension: temperature layer
                                                       (see below)
                   second dimension: transmittance layer
  dqtau
          - real array (of size maxlev) of derivatives of
                 tau with respect to changes in moisture at given layers
                  -these apply to taus at all layers below the given layer
          - real array (of size maxlev) of derivatives of
  dutau
                 tau with respect to changes in ozone at given layers
                  -these apply to taus at all layers below the given layer
  dtsun
          - real array (of size maxlev) of derivatives of
              suntau with respect to changes in temperature at given layers
! dqsun
          - real array (of size maxlev) of derivatives of
              suntau with respect to changes in moisture at given layers
 dusun
          - real array (of size maxlev) of derivatives of
              suntau with respect to changes in ozone at given layers
```

```
!!OUTPUT PARAMETERS:
            - real brightness temp for channel chan calculated from profile
            - real array (of size maxlev) of d(Tb)/dT
  dTb_dT
            - real array (of size maxlev) of derivative of brightness
  dTb_dq
                 temperature with respect to moisture at given layers
            - real array (of size maxlev) of derivative of brightness
 dTb_du
                 temperature with respect to ozone at given layers
!!REVISION HISTORY:
  20Dec95 Meta S. original routine from combining dTb/dT(simple) and
                   dTb/dq2 modules
  02jan96 Meta S. change to calculate values for 1 channel, call
                    from tovs_tauK routine
  06Jan96 Meta S. modify to use tau-level dependent derivatives for
                      temperature (dttau), to include C term in
                       fixed gas temperature derivative.
  12Jan96 Meta S. edited prologue
  29mar96 Meta S. minor optimization - change dttau to packed array
                      format
! Matrix Packing
                                (based on SPPTRF manual page)
! The j-th column of A is stored in the array AP as follows:
  AP(i + (j-1)*j/2) = A(i,j) for 1<=i<=j
     Two-dimensional storage of the matrix A:
                         1st dimension: affecting temperature layer
       a11 a12 a13 a14
        0 a22 a23 a24
                         2nd dimension: affected transmittance layer
          0 a33 a34
        0
          0 0 a44 a12: influence of T(1) on hat(tau)(2)
        | zeros because temperatures in a given layer don't affect
             transmittance layers above that layer
    Packed storage of the upper triangle of A:
    AP = [a11, a12, a22, a13, a23, a33, a14, a24, a34, a44]
subroutine dTb_dTQU(tau,tausun,chan,tair,tmptop,ir_zen,
    &
          emissmw, land, nlev, rho, secsun, dosun, tground,
          tsurfair, dttau, dqtau, dutau, dtsun, dqsun, dusun,
          bt, dTb_dT, dTb_dq, dTb_du)
     implicit none
     include "tovsparam.h"
```

```
integer maxpack
     parameter (maxpack = (maxlev*(maxlev+1))/2 )
! input variables
     real tau(maxlev)
     real tausun
     integer chan
     real tair(maxlev)
     real tmptop
     real ir_zen
     real emissmw
     real rhosec
     logical land
     integer nlev
     real rho
     real secsun
     logical dosun
     real tground
     real tsurfair
     real dttau(maxpack)
     real dqtau(maxlev)
     real dutau(maxlev)
     real dtsun(maxlev)
     real dqsun(maxlev)
     real dusun(maxlev)
! output variables
     real bt
     real dTb_dT(maxlev)
     real dTb_dq(maxlev)
     real dTb_du(maxlev)
! parameters from common
     include "phycons.h"
     include "chacons.h"
     include "matcons.h"
! function types
     real planck
     real dplanck
! local variables
                              ! blackbody temp. of avg layer temperature
     real btav
     real
                              ! blackbody temp. of surface air temperature
               btsurf
               btgnd
                              ! blackbody temp. of ground temperature
     real
     real
               calc_rad
                              ! calculated radiance
     real
                dbtav
                              ! d(btav) / dT
                              ! difference of transmittance in layer
     real
                deltau
     real
                dtauinv
                              ! inverse transmittance difference
                dRd_dTav(maxlev) ! d (Rdown)/ d Tavg
     real
                               ! d (Rdown)/ d H2O
     real
                dRd_dq(maxlev)
                dRd_du(maxlev)
                                  ! d (Rdown)/ d ozone
     real
                dTb_dTav(maxlev) ! d (bright. T)/ d Tavg
     real
```

```
real
               dTb dR
                                ! d (bright. T)/ d (radiance)
     real
               emiss
                                 ! sfc emissivity for this sounding
                                ! counters
     integer
               1, m, n, nm1
     real
               pcon1
     real
               pcon2
                                ! atmos. emission downward flux
     real
               rad_down
                                ! solar radiation at TOA (?)
     real
               rad_sun
                                ! solar contribution to calc. radiance
     real
               sunterm
     real
               tav(maxlev)
                                ! mean layer temperature
! check the following to see if using double makes a difference
     double precision factor, dqfactor, dufactor, dtfactor
     pcon1 = planck1(chan)
     pcon2 = planck2(chan)
 initialize arrays
     do l = 1, maxlev
       dTb_dTav(1) = 0.
       dRd_dTav(1) = 0.
                            ! derivative from rad_down; work array
       dTb_dq(1) = 0.0
       dTb_du(1) = 0.0
       dRd_dq(1) = 0.0
       dRd_du(1) = 0.0
     enddo
     tav(1) = 0.5 * (tair(1) + tmptop)
     do 1 = 2, nlev
       tav(1) = 0.5 * (tair(1) + tair(1-1))
     enddo
ŗ
     if (irchan(chan)) then
Ţ
! initialize radiance sum
        deltau = 1.0 - tau(1)
        btav = planck(tav(1), pcon1, pcon2)
        calc_rad = btav * deltau
! derivs wrt tau
        dTb_dq(1) = dTb_dq(1) - btav * tau(1) * dqtau(1)
        dTb_du(1) = dTb_du(1) - btav * tau(1) * dutau(1)
        dTb_dTav(1) = dTb_dTav(1) - btav * tau(1) * dttau(1)
! derivs wrt. planck function
        dbtav = dplanck(tav(1), pcon1, pcon2)
        dTb_dTav(1) = dTb_dTav(1) + dbtav * deltau
  integrate through atmosphere (from top down)
        do 1 = 2, nlev
```

```
n = (1-1)*1/2
                                ! set index for packed array
           nm1 = (1-2)*(1-1)/2! for levels 1 and 1-1
           btav = planck(tav(1), pcon1, pcon2)
           deltau = tau(l-1) - tau(l)
           calc_rad = calc_rad + btav * deltau
! derivative terms at 1 and for levels above 1 (which contribute to tau)
! first, derivative for current layer (level) 'l'
           dTb_dq(1) = dTb_dq(1) - btav * tau(1) * dqtau(1)
           dTb_du(1) = dTb_du(1) - btav * tau(1) * dutau(1)
           dTb_dTav(1)=dTb_dTav(1) - btav * tau(1) * dttau(1+n)
! then, derivatives for layers above 'l' (which contribute to both
    tau(1) and tau(1-1)) - loop over temperature, constituent layers
     to add contribution for current layer of vertical integral
           do m = 1,1-1
             dTb_dq(m) = dTb_dq(m) + btav * deltau * dqtau(m)
             dTb_du(m) = dTb_du(m) + btav * deltau * dutau(m)
             dTb_dTav(m) = dTb_dTav(m) + btav *
    &
                (tau(l-1) * dttau(m+nm1) -
    &r.
                  tau(1) * dttau(m+n) )
           enddo
! then derivs wrt. planck function
           dbtav = dplanck(tav(1),pcon1,pcon2)
           dTb_dTav(l) = dTb_dTav(l) + dbtav * deltau
         enddo
 prepare to calculate surface contribution
        btsurf = planck(tsurfair, pcon1, pcon2)
        btgnd = planck(tground, pcon1, pcon2)
        rad_down = dfxcon(chan) * btsurf * (1. - tau(nlev))
        n = (nlev-1)*nlev/2
                                   ! index for nlev
! calculate derivative of rad_down wrt. changes in t,q,u at levels above
      (derivatives of tau(nlev) wrt changes in tau above)
        do m = 1,nlev
           dRd_dq(m) = -dfxcon(chan) * btsurf * tau(nlev)
    &r.
                                * dqtau(m)
           dRd_du(m) = -dfxcon(chan) * btsurf * tau(nlev)
                                * dutau(m)
    &
           dRd_dTav(m) = -dfxcon(chan) * btsurf * tau(nlev)
                                * dttau(m+n)
    &
        enddo
        if (land) then
```

```
emiss = em_land(chan)
        else
           emiss = em_water(chan)
         endif
      else if (mwchan(chan)) then
! initialize radiance sum
        btav = tav(1)
        deltau = 1.0 - tau(1)
        if (tau(1) .ne. 0.0) then
           dtauinv = (1. / tau(1)) - 1.
           rad_down = dtauinv * btav
           dRd_dTav(1) = dRd_dTav(1) + dtauinv
        endif
        calc_rad = btav * deltau
! first, deriv wrt. tau
        dTb_dq(1) = dTb_dq(1) - btav * tau(1) * dqtau(1)
        dTb_du(1) = dTb_du(1) - btav * tau(1) * dutau(1)
        dTb_dTav(1) = dTb_dTav(1) - btav * tau(1) * dttau(1)
! for derivative, note that "dqtau", "dutau" are
    1/tau dtau/dg 1/tau dtau/du - so need to multiply by tau
       (i.e. not divide by tau^2 but only by tau)
        if (tau(1) .ne. 0.0) then
           dRd_dq(1) = dRd_dq(1)
                       - btav * 1./tau(1) * dqtau(1)
    &
           dRd_du(1) = dRd_du(1)
                       - btav * 1./tau(1) * dutau(1)
    &
           dRd_dTav(1) = dRd_dTav(1)
                       - btav * 1./tau(1) * dttau(1)
    &
         endif
! now add contribution from deriv wrt. temperature
        dTb_dTav(1) = dTb_dTav(1) + deltau
! integrate through atmosphere (from top down)
        do 1 = 2, nlev
           n = (1-1)*1/2
                                ! set index for packed array
           nm1 = (1-2)*(1-1)/2 ! for levels 1 and 1-1
           btav = tav(1)
           deltau = tau(l-1) - tau(l)
           if ((tau(1) .ne. 0.0) .and.
                (tau(1-1) .ne. 0.0)) then
              dtauinv = (1. / tau(1)) - (1. / tau(1-1))
              rad_down = rad_down + dtauinv * btav
           calc_rad = calc_rad + btav * deltau
```

```
! tau derivative terms at 1 and for levels above 1 (which contribute to tau)
! first, tau derivative for current layer (level) 'l'
           dTb_dq(1) = dTb_dq(1) - btav * tau(1) * dqtau(1)
           dTb_du(1) = dTb_du(1) - btav * tau(1) * dutau(1)
           dTb_dTav(1) = dTb_dTav(1) -
    &
                                 btav * tau(l) * dttau(l+n)
! then, tau derivatives for layers above 'l' (which contribute to both
    tau(1) and tau(1-1) -- loop over temperature, constituent layers
     to add contribution for current layer of vertical integral
           do m = 1.1-1
            dTb_dq(m) = dTb_dq(m) + btav * deltau * dqtau(m)
            dTb_du(m) = dTb_du(m) + btav * deltau * dutau(m)
            dTb_dTav(m) = dTb_dTav(m) + btav *
               (tau(l-1) * dttau(m+nm1) -
    &
                 tau(1) * dttau(m+n) )
    &
           enddo
! then, tau derivatives for rad_down terms (if applicable)
! for derivative, note that "dqtau", "dutau" are
    1/tau dtau/dq 1/tau dtau/du - so need to multiply by tau
      (i.e. not divide by tau^2 but only by tau)
           if ((tau(1) .ne. 0.0) .and.
    &r.
               (tau(l-1) .ne. 0.0)) then
! tau derivative for current layer 'l'
             dRd_dq(1) = dRd_dq(1)
                      - btav * 1./tau(1) * dqtau(1)
    &
             dRd_du(1) = dRd_du(1)
                      - btav * 1./tau(1) * dutau(1)
    &.
             dRd_dTav(1) = dRd_dTav(1)
                      - btav * 1./tau(1) * dttau(1+n)
    &
! then, tau derivatives for layers above 'l'... note the tau derivative terms
      in "dtauinv" are the same for each level (needn't be recalculated)
             do m = 1, 1-1
                dRd_dq(m) = dRd_dq(m) -
    &r.
                        btav * dtauinv * dqtau(m)
                dRd_du(m) = dRd_du(m) -
                        btav * dtauinv * dutau(m)
    &
                dRd_dTav(m) = dRd_dTav(m) - btav *
                    (1./tau(1) * dttau(m+n) -
    &
    &
                     1./tau(l-1) * dttau(m+nm1)
             enddo
  then, contribution from temperature term to rad_down derivative
             dRd_dTav(1) = dRd_dTav(1) + dtauinv
```

```
endif
```

```
! finally, add contribution from temperature term to Tb derivative
            dTb_dTav(1) = dTb_dTav(1) + deltau
         enddo
! prepare to calculate surface contribution
        btsurf = tsurfair
         btgnd = tground
        rad_down = (rad_down + bigbang) * tau(nlev)
         emiss = emissmw
        n = (nlev-1)*nlev/2
                             ! index for nlev
        do m = 1, nlev
            dRd_dq(m) = dRd_dq(m) * tau(nlev) +
     &
                             rad_down * dqtau(m)
            dRd_du(m) = dRd_du(m) * tau(nlev) +
                             rad_down * dutau(m)
     &
            dRd_dTav(m) = dRd_dTav(m) * tau(nlev) +
                             rad_down * dttau(m+n)
     &r.
         enddo
      endif
      if (use_refl(chan)) then
         factor =
           (cos(ir_zen*deg2rad)*calc_rad*(1.-emiss)*(dfxcon(chan)*2.0)
     &
              + emiss * btgnd)
         calc_rad = calc_rad + tau(nlev) * factor
         n = (nlev-1)*nlev/2
                                 ! index for nlev
         do m = 1, nlev
                       (cos(ir_zen*deg2rad)*dTb_dq(m) *
            dqfactor =
                            (1.-emiss) * (dfxcon(chan)*2.0)
     &
            dTb_dq(m) = dTb_dq(m) +
     &
                 tau(nlev) * dqtau(m) * factor +
                 tau(nlev) * dqfactor
     &
            dufactor = (cos(ir_zen*deg2rad)*dTb_du(m) *
                            (1.-emiss) * (dfxcon(chan)*2.0)
     &
            dTb_du(m) = dTb_du(m) +
                 tau(nlev) * dutau(m) * factor +
     &
                 tau(nlev) * dufactor
     &
            dtfactor = (cos(ir_zen*deg2rad)*dTb_dTav(m) *
                            (1.-emiss) * (dfxcon(chan)*2.0)
     &
            dTb_dTav(m) = dTb_dTav(m) +
```

```
tau(nlev) * dttau(m+n) * factor +
    &r.
                 tau(nlev) * dtfactor
    &
         enddo
      else
         calc_rad = calc_rad + tau(nlev) *
              (rad_down + emiss * ( btgnd - rad_down ))
    &
         n = (nlev-1)*nlev/2
                                   ! index for nlev
        factor = (rad_down + emiss * ( btgnd - rad_down ))
        do m = 1, nlev
            dTb_dq(m) = dTb_dq(m) +
    &
                tau(nlev) * dqtau(m) * factor +
                tau(nlev) * (1.-emiss) * dRd_dq(m)
     &
            dTb_du(m) = dTb_du(m) +
                tau(nlev) * dutau(m) * factor +
     &
                tau(nlev) * (1.-emiss) * dRd_du(m)
     &
            dTb_dTav(m) = dTb_dTav(m) +
                tau(nlev) * dttau(m+n) * factor +
    &
                tau(nlev) * (1.-emiss) * dRd_dTav(m)
    &
         enddo
      endif
! Add solar contribution for wavenumber greater than 2000.0
      if (dosun) then
        rhosec = rho / secsun
         rad_sun = scon * planck(tsun, pcon1, pcon2)
         sunterm = rhosec * rad_sun * tausun
        calc_rad = calc_rad + sunterm
         do m = 1, nlev
            dTb_dq(m) = dTb_dq(m) + rhosec * rad_sun * dqsun(m)
            dTb_du(m) = dTb_du(m) + rhosec * rad_sun * dusun(m)
            dTb_dTav(m) = dTb_dTav(m) + rhosec * rad_sun * dtsun(m)
         enddo
      endif
! now calculate brightness temperature & derivative
      if (irchan(chan)) then
         call brtemp(calc_rad, pcon1, pcon2, bt)
         dTb_dR = 1.0 / dplanck(bt,pcon1,pcon2)
         do m = 1, nlev
           dTb_dq(m) = dTb_dq(m) * dTb_dR
          dTb_du(m) = dTb_du(m) * dTb_dR
           dTb_dTav(m) = dTb_dTav(m) * dTb_dR
         enddo
```

```
else if (mwchan(chan)) then
!
! no dTb/dR factor for microwave channels

    bt = calc_rad

    endif
! now, calculate dTb/dT = dTb_dTav * dTav/dT

    do l = 1,nlev-1
        dTb_dt(l) = 0.5 * ( dTb_dTav(l) + dTb_dTav(l+1))
    enddo
    dTb_dt(nlev) = 0.5 * dTb_dTav(nlev)

    return
    end
```

tovsfix_tauK.f - Fixed gas transmittance contribution

```
!!TOVSFIX TAUK
!!ROUTINE: tovsfix_tauK
!!DESCRIPTION: Calculate contribution to transmittance by fixed gases
              using rapid transmittance algorithm coefficients
              - reference Susskind, et al. 1983 JGR, p 8565
              and derivative of fixed contrib w.r.t. mean layer temperature
! CALLED FROM: tovs_tauK
! SYSTEM ROUTINES USED: none
! SUBROUTINES CALLED: none
!!INPUT PARAMETERS:
! tdif
         - real array (of length nlev) with difference between
              current best estimate of layer mean temperatures and
              standard atmosphere layer mean temperatures (K)
          - integer channel number
! chan
          - integer surface level (also number of levels to use)
 nlev
!!OUTPUT PARAMETERS:
! taufix - real array of effective mean layer transmittance
             for fixed gases
  dttaufix - real array (of size maxpack by maxangs) of
                 derivative of taufix with respect to temperature
                 ( 1/taufix d(taufix)/dT)
                   first dimension: temperature layer
                                                       (see below)
 packed matrix:
                   second dimension: transmittance layer
!!REVISION HISTORY:
  17jan96 Meta S.
                   original routine derived from module tovsfix_tau
                   in GLA retrieval code - move derivative code into
                   this routine, and vertical sum from tovs_tauprodK
                   back into tovs_tauangK
  16feb96 Meta S. minor optimization: swap dimensions of dttaufix
  29mar96 Meta S. another minor optimization: change dttaufix
                   into packed array ((maxlev*(maxlev+1))/2,maxangs)
! 18may96 Meta S. effTdif need not be saved as fct. of angle
! Matrix Packing
                               (based on SPPTRF manual page)
! The j-th column of A is stored in the array AP as follows:
  AP(i + (j-1)*j/2) = A(i,j) \text{ for } 1 \le i \le j
     Two-dimensional storage of the matrix A:
į
       all al2 al3 al4   1st dimension: affecting temperature layer
```

```
0 a22 a23 a24
                         2nd dimension: affected transmittance layer
        0 0 a33 a34
        0 0 0 a44
                         a12: influence of T(1) on \hat(\tau)(2)
        | zeros because temperatures in a given layer don't affect
             transmittance layers above that layer
    Packed storage of the upper triangle of A:
    AP = [a11, a12, a22, a13, a23, a33, a14, a24, a34, a44]
subroutine tovsfix_tauK(tdif, chan, nlev, taufix, dttaufix)
     implicit none
     include "tovsparam.h"
     integer maxpack
     parameter (maxpack = (maxlev*(maxlev+1))/2 )
  variables passed into routine
     integer nlev, chan
     real tdif(maxlev)
 variables output from routine
     real taufix(maxlev,maxangs)
     real dttaufix(maxpack,maxangs)
! constants, quasi-constants, and coefficients
     include "taucoef.h"
! local variables
     integer k, l, m, n
! difference of effective mean temperature above given level
   from rapid algorithm standard temperature value
     real effTdif
     real cltau
! Susskind et al 1983 algorithm for transmittance from fixed gases
! Effective mean temperature above P -> \tilde{T} in equation (A6)
! taucfw: precomputed non-varying part of \tilde{T} integral
  taucfc: includes term with division by (1 - \tau^o)
    here effTdif -> \tilde{T} - \tilde{T}^o or
      difference between effective mean temperatures for the
      current temperature profile and the standard temperature profile
     tau = A + B (T - T^o) + C ( \tilde{T} - \tilde{T}^o)
                                                           (A7)
```

```
do k = 1, maxangs
         effTdif = 0.
         do l = 1, nlev
           n = (1-1)*1/2
                              ! set index for packed array
           effTdif = effTdif + taucfw(1,k,chan) * tdif(1)
           taufix(1,k) = taucfa(1,k,chan) + taucfb(1,k,chan) * tdif(1)
                 + taucfc(l,k,chan) * effTdif
! layer transmittance bounded between 0 and 1
           taufix(l,k) = amax1(taufix(l,k), 0.0)
           taufix(l,k) = amin1(taufix(l,k), 1.0)
! if taufix is zero, then set temperature derivative also to zero
            if (taufix(1,k) .eq. 0.0) then
               do m = 1,1
                  dttaufix(m+n,k) = 0.0
               enddo
            else
! effTdif is weighted sum of temperatures over all layers above
  current layer - so need to loop over those layers to fill in
   their contribution to this layer's effective transmittance
               cltau = taucfc(l,k,chan) / taufix(l,k)
               dom = 1.1
                  dttaufix(m+n,k) = cltau * taucfw(m,k,chan)
               enddo
! temperature contribution in 'taucfb' term is only for transmittance
! in the same layer
               dttaufix(l+n,k) = dttaufix(l+n,k) +
    &
                  (taucfb(l,k,chan) / taufix(l,k))
            endif
         enddo
        do l = nlev+1,maxlev
           n = (1-1)*1/2
                              ! set index for packed array
           taufix(l,k) = 0.0
į
           do m = 1,1
į
               dttaufix(m+n,k) = 0.0
           do m = n+1, n+1
               dttaufix(m,k) = 0.0
           enddo
         enddo
      enddo
     return
      end
```

tovsh2o_tauK.f - Water vapor transmittance contribution

```
!!TOVSH2O TAUK
!!ROUTINE: tovsh2o_tauK
!!DESCRIPTION: Calculate contribution to transmittance by water vapor
              using rapid transmittance algorithm coefficients
              and derivatives with respect to H2O and temperature
              - reference Susskind, et al. 1983 JGR, p 8564
! CALLED FROM: tovs_tauK
! SYSTEM ROUTINES USED: exp
! SUBROUTINES CALLED: none
!!INPUT PARAMETERS:
! h2ocd - real array (of length maxlev) with water vapor layer
               column densities (g cm^-2)
          - real array (of length maxlev) of H2O vapor pressure (atm)
  dqh2ocd - real array (of length maxlev) of derivative of H2O
              column density with respect to specific humidity
  dqe
          - real array (of length maxlev) of derivative of H2O
              vapor pressure with respect to specific humidity
          - integer channel number
          - integer surface level (also number of levels to use)
 nlev
  tav - real array (of length maxlev) with temperature profile (K)
  celsius - real array (of length maxlev) with (T - 273) (approx.
                 temperature in celsius)
!!OUTPUT PARAMETERS:
  tauh2o - real array (of size maxlev by maxangs) of effective h2o
                 transmittance
 dqtauh2o - real array (of size maxlev by maxangs) of logarithmic
                 derivative of tauh2o with respect to H2O specific humidity
 dttauh2o - real array (of size maxlev by maxangs) of logarithmic
                 derivative of tauh2o with respect to temperature
!!REVISION HISTORY:
  30nov95 Meta S. original routine derived from module tovsh2o_tau
  07dec95 Meta S. added loop to fill in zeros below sfc level for
                      derivative
  19dec95 Meta S.
                   add 'factor' in calculation of 'cont', 'dqcont'
                   (fix bug where 'cont' was reset before being used
                      in calculation of 'dqcont')
  20Dec95 Meta S. this version calculates temperature derivative
                     wrt 'tav' (simplifies coding)
 15Mar96 Meta S. amplified comments - tiny fix with 'dtcont'
  18may96 Meta S. revisions to move column density and vapor pressure
                    calculations up to tovs_tauK
```

```
! 20Jul96 Meta S. corrected description of output parameters
subroutine tovsh2o_tauK( h2ocd, e, dqh2ocd, dqe, chan, nlev,
              tav, celsius, tauh2o, dqtauh2o, dttauh2o)
    &
     implicit none
     include "tovsparam.h"
! variables passed into routine
     real
           h2ocd(maxlev)
     real
             e(maxlev)
     real
            dqh2ocd(maxlev)
             dqe(maxlev)
     real
     integer chan
     integer nlev
     real
             tav(maxlev)
     real
             celsius(maxlev)
! variable output from routine
             tauh2o(maxlev,maxangs)
     real
     real
             dqtauh2o(maxlev,maxangs)
     real
             dttauh2o(maxlev,maxangs)
! constants, quasi-constants, and coefficients
     include "taucoef.h"
     include "chacons.h"
     include "grdcons.h"
! local storage
     real
            cont(maxlev)
             deffh2o(maxlev)
     real
     real
            dqcont(maxlev)
             dtcont(maxlev)
     real
             dqtaulog
     real
     real
             dttaulog
             effh2o(maxlev)
     real
     integer k, l
     real
             taulog
! statement function definitions
     real
             quikexp
     real
     quikexp(X) = 1.0 + X * (1.0 + X * (.5 + X * .16666667))
! for IR channel, calculate continuum (except for angle contribution)
```

```
! and derivatives with respect to specific humidity and temperature
    see Susskind and Searl (1978) J. Quant. Spect. Radiat. Transfer
  kH2O(\nu,i) = \sum_{k=0}^{\infty} k2(\nu,T) e(1) U(1)
  U(1) (as above) --> h2ocd = delprs * h2omid
  degchn = - dk2/dT for channel k2(T) = k2(contmp)*(1-degchn)(T-contmp)
      if (irchan(chan)) then
         do l = 1, nlev
            cont(1) = e(1) * abscof(chan) * h2ocd(1) *
                 (1.0 - degchn(chan) * (tav(l) - contmp(chan)))
    &
            dtcont(1) = -e(1) * abscof(chan) * h2ocd(1) *
                degchn(chan)
    &
            dqcont(1) = abscof(chan) *
     &
                   (e(1) * dqh2ocd(1) + h2ocd(1) * dqe(1)) *
                 (1.0 - degchn(chan) * (tav(1) - contmp(chan)))
     Ŋ۲.
         enddo
      else
         do l = 1, nlev
            cont(1) = 0.0
            dqcont(1) = 0.0
            dtcont(1) = 0.0
         enddo
      endif
      if (taucfm(chan).lt. 0.99) then
         do l = 1, nlev
            deffh2o(1) = taucfm(chan) * (h2ocd(1)**(taucfm(chan)-1.))
     &
                    * dqh2ocd(1)
            effh2o(1) = h2ocd(1) ** taucfm(chan)
         enddo
      else
         do l = 1, nlev
            effh2o(1) = h2ocd(1)
            deffh2o(1) = dqh2ocd(1)
         enddo
      endif
  From (A5) in Susskind, et al. 1983 (page 8564)
! tau(H2O) = exp( - (D(1 - E(T - 273)) h2ocd**M)
          { * exp(-sec \theta * KcontinuumH2O) of course }
      see also ON-9608, eq32
```

```
From GLA retrieval code:
    TAULOG(II,K) = (-1.0 - TAUCFE(K,YCHA) * TMPXCES(II))
                    * TAUCFD(LEVL,K,YCHA) * EFFH2O(II)
!
!
    so perhaps TAUCFE = -E
Ţ
     do k = 1, numangs
         do l = 1, nlev
            taulog = ( -1.0 - taucfe(k,chan) * celsius(l))
                 * taucfd(1,k,chan) * effh2o(1)
           taulog = taulog - secang(k) * cont(l)
           taulog = amin1(taulog, 0.0) !<-- modified for derivative calc.
ŗ
            if (taulog .gt. 0.0) then
!
              dqtauh2o(1,k) = 0.0
                                       ! if taulog set to zero, derivatives
             dttauh2o(1,k) = 0.0
                                       ! should be zero also
į
              taulog = 0.0
              tauh2o(1,k) = 1.0
Ţ
            else
  set derivatives w.r.t. ozone and temperature of mean layer transmittance
              dqtaulog = ( -1.0 - taucfe(k,chan) * celsius(l))
    &r.
                 * taucfd(1,k,chan) * deffh2o(1)
              dqtaulog = dqtaulog - secang(k) * dqcont(1)
              dttaulog = - taucfe(k,chan)
                 * taucfd(1,k,chan) * effh2o(1)
    &
              dttaulog = dttaulog - secang(k) * dtcont(l)
              tauh2o(1,k) = quikexp( taulog )
  note on derivatives: the factor exp(taulog) is not included here
                          since we would divide by exp(taulog)
                          when creating full tau derivative
  we are calculating logarithmic derivative here ->
   1/tauh2o d(tauh2o)/dq and 1/tauh2o d(tauh2o)/dt
              dqtauh2o(1,k) = dqtaulog
              dttauh2o(1,k) = dttaulog
            endif
         enddo
        do l = nlev+1,maxlev
           dqtauh2o(1,k) = 0.0
           dttauh2o(1,k) = 0.0
            tauh2o(l,k) = 1.0! shouldn't be used, anyway
         enddo
      enddo
     return
      end
```

tovsozo_tauK.f - Ozone transmittance contribution

```
!!TOVSOZO TAUK
!!ROUTINE: tovsozo_tauK
!!DESCRIPTION: Calculate contribution to transmittance by ozone
             using rapid transmittance algorithm coefficients
             - reference Susskind, et al. 1983 JGR, p 8564
             and derivatives with respect to ozone and temperature
! CALLED FROM: tovs_tauK
! SYSTEM ROUTINES USED: exp
! SUBROUTINES CALLED: none
!!INPUT PARAMETERS:
! ozomid - real array (of length mlev) with ozone column
                densities (g/cm**2)
! chan
          - integer channel number
          - integer number of levels (top to surface)
! celsius - real array (of length mlev) with profile of (T-273) (K)
!!OUTPUT PARAMETERS:
! tauozo - real array (of size maxlev by maxangs) of effective
           ozone transmittance
 dutauozo - real array (of size maxlev by maxangs) of derivative
                of tauozo with respect to ozone
 dttauh2o - real array (of size maxlev by maxangs) of derivative
                of tauh2o with respect to temperature
!!REVISION HISTORY:
! Oldec95 Meta S. original routine derived from module tovsozo_tau
 11dec95 Meta S. bug fixes: some typos (using H2O coeffs for deriv.
                   instead of O3 coeffs)
! 20Dec95 Meta S. this version calculates temperature derivative
                    wrt 'tav' (simplifies coding)
 14Mar96 Meta S. added more comments
! 10Jul96 Meta S. corrected description of output parameters
subroutine tovsozo_tauK(ozomid, chan, nlev, celsius, tauozo,
                dutauozo, dttauozo)
     implicit none
     include "tovsparam.h"
! variables passed into routine
```

```
ozomid(maxlev)
      real
      integer chan
      integer nlev
               celsius (maxlev)
      real
  variables output from routine
               tauozo(maxlev,maxangs)
     real
     real
               dttauozo(maxlev, maxangs)
     real
               dutauozo(maxlev, maxangs)
! constants, quasi-constants, and coefficients
      include "taucoef.h"
! local storage
               deffO3(maxlev)
      real
     real
               dttaulog
     real
               dutaulog
               eff03(maxlev)
     real
      integer k, l
     real
               taulog
! statement function definitions
     real
               quikexp
     real
               χ
      quikexp(X) = 1.0 + X * (1.0 + X * (.5 + X * .16666667))
  calculate 'eff03' as (ozone conc.) ** taucfN
  following is formula used in GLA code:
       if (taucfn(chan) .1t. 0.99) then
          do l = 1, nlev
             eff03(1) = taucfn(chan) * alog( ozomid(1))
             eff03(1) = exp(eff03)
          enddo
       else
          do l = 1, nlev
             effO3(1) = ozomid(1)
          enddo
į
       endif
  deff03 is derivative of eff03 with respect to ozone
      do l = 1, nlev
         eff03(1) = ozomid(1) ** taucfn(chan)
         deff03(1) = taucfn(chan) * (ozomid(1)**(taucfn(chan)-1.))
      enddo
 From (A5) in Susskind, et al. 1983 (page 8564)
! tau(ozone) = exp( - (F(1 - G(T - 273)) ozomid**N)
```

```
see also ON-9608 eq 33
! from GLA retrieval code:
! TAULOG(II,K) = (-1.0 - TAUCFG(K,YCHA) * TMPXCES(II))
   * TAUCFF(LEVL,K,YCHA) * EFFO3(II) * TAUCFF(LEVL,K,YCHA) * EFFO3(II)
 so sign of TAUCFG = -G , perhaps...
     do k = 1, numangs
        do l = 1, nlev
            taulog = (-1.0 - taucfg(k, chan) * celsius(l))
                 * taucff(1,k,chan) * eff03(1)
    &
i
            taulog = amin1 (taulog, 0.0) !<-- modified for derivative calc.
            if (taulog .gt. 0.0) then
              dutauozo(1,k) = 0.0
                                       ! if taulog set to zero, derivatives
              dttauozo(1,k) = 0.0
                                       ! should be zero also.
!
              taulog = 0.0
              tauozo(1,k) = 1.0
            else
  set derivatives w.r.t. ozone and temperature of mean layer transmittance
              dutaulog = ( -1.0 - taucfg(k,chan) * celsius(l))
                 * taucff(1,k,chan) * deff03(1)
    &
              dttaulog = -taucfg(k,chan)
                 * taucff(1,k,chan) * deff03(1)
              tauozo(l,k) = quikexp( taulog )
  note on derivatives: the factor exp(taulog) is not included here
                          since we would divide by exp(taulog)
                          when creating full tau derivative
  we are calculating logarithmic derivative 1/tauozo d(tauozo)/dT
    and 1/(tauozo) d(tauozo)/du
              dutauozo(1,k) = dutaulog
              dttauozo(1,k) = dttaulog
            endif
         enddo
         do l = nlev+1,maxlev
           dutauozo(1,k) = 0.0
           dttauozo(1,k) = 0.0
           tauozo(l,k) = 1.0
                                   !shouldn't be used, anyway
         enddo
      enddo
     return
      end
```

tovs_tauK.f - Driver for transmittance derivative calculation

```
!!TOVS TAUK
!!ROUTINE: tovs_tauK
!!DESCRIPTION: Calculate transmittance for given channel numbers
              for a specified temperature, moisture, and ozone profile
              and derivatives with respect to temperature, moisture
              and ozone to be used in Jacobian calculation
              adding in angle calculation
              Based on module tovs_tau
! SYSTEM ROUTINES USED: abs, cos
! SUBROUTINES CALLED:
     tovsfix_tauK - transmittance contrib. by fixed gases and derivative
     tovsh2o_tauK - transmittance contrib. by water vapor and derivative
     tovsozo_tauK - transmittance contrib. by ozone and derivative
ŗ
     tovs_tauprodK - take product of fixed, water vapor & ozone
                      contribs and derivatives
     tovs_tauangK - interpolate transmittance to obs. zenith angle &
                        calculate consistent derivative terms
                 - calculate brightness temperature and Jacobian
     dTb_dTQU
!!INPUT PARAMETERS:
! tair - real array (of length maxlev) with temperature profile (K)
! tmptop - real temperature (K) at "top of atmosphere"
! h2omid - real array (of length maxlev) with water vapor layer specific
              humidities (KG/KG)
! ozomid - real array (of length maxlev) with ozone column densities
!
               (g/cm**2)
         - real surface pressure (mb)
! ps
! nchan - integer number of channels to calculate
! chanarr - integer array (of length maxcha) of channels to calculate
! nlev - integer surface level (also number of levels to use)
! mw_zen - real
                    satellite zenith angle for microwave channels
                    satellite zenith angle for infrared channels
! ir_zen - real
! dayflag - logical true if day ( sun angle > SUNSET = 85.0E0)
! secsun - real 1. / cos( sun angle)
  emissmw - real microwave surface emissivity
! land - logical flag (.true. if land, .false. if water)
! rho - real bidirectional reflectance
 tground - real skin (ground) temperature (K)
! tsurfair - real surface air temperature (K)
!!OUTPUT PARAMETERS:
! tau
        - real array (of size maxlev by maxcha) of transmittance along
```

```
path to satellite.
  tausun - real array (of size maxcha) of transmittance along path from
            - real array (of size maxcha) of brightness temperatures for
                 for channels in chanarr, calculated from profile
           - real array (of size maxlev by maxcha) of d(Tb)/dT
  dTbdT
                 derivative of brightness temperature with respect to
                 temperature at given levels
  dTbdq
           - real array (of size maxlev by maxcha) of derivative
                  of brightness temperature with respect to specific
                 humidity at given layers
  dTbdu
              real array (of size maxlev by maxcha) of derivative
                  of brightness temperature with respect to ozone at
                  given layers
  errflag - logical returns .true. if error
!!REVISION HISTORY:
! O4dec95 Meta S. original routine derived from subroutine tovs_tau
  07dec95 Meta S.
                   carry derivatives through to tovs_tauNK
  15dec95 Meta S.
                   replace non-interpolated tovs_tauNK with
                       interpolating tovs_tauangK
  20dec95 Meta S.
                   add changes to create temperature derivatives
  02jan96 Meta S.
                   fold in rest of jacobian calculation - put in call
                       to dTb\_dTQU2
  08jan96 Meta S.
                   changes for C term dependence of temperature
  29mar96 Meta S.
                   minor optimization: 1-d packed array for temperature
                    derivative terms
  18may96 Meta S.
                   moved calculation of column H2O density and vapor
                    pressure from tovsh2o_tau into this routine
subroutine tovs_tauK(tair, tmptop, h2omid, ozomid, ps, nchan,
          chanarr, nlev, mw_zen, ir_zen, dayflag, secsun,
          emissmw, land, rho, tground, tsurfair,
    &
          tau, tausun, bt, dTbdT, dTbdq, dTbdu, errflag)
     implicit none
     include "tovsparam.h"
     integer maxpack
     parameter (maxpack = (maxlev*(maxlev+1))/2 )
  variables passed into routine
     real
              tair(maxlev)
     real
              tmptop
              h2omid(maxlev)
     real
              ozomid(maxlev)
     real
     real
              ps
     integer nchan
     integer chanarr(maxcha)
```

```
integer nlev
      real mw_zen
      real
              ir_zen
      logical dayflag
      real
             secsun
      real
              emissmw
      logical land
      real rho
      real
             tground
             tsurfair
      real
! variables output from routine
     real tau(maxlev, maxcha)
real tausun(maxcha)
real bt(maxcha)
real dTbdT(maxlev,maxcha)
real dTbdq(maxlev,maxcha)
real dTbdu(maxlev,maxcha)
      logical errflag
! constants, quasi-constants, and coefficients
      include "matcons.h"
      include "grdcons.h"
      include "chacons.h"
      include "phycons.h"
! local variables
      integer 1
                                      ! counters
      integer ichan
      integer chan
      real tav(maxlev)
                                     ! mean layer temperature
      real tdif(maxlev)
                                     ! difference from rapid alg std. temp.
                                      ! difference from 273
      real celsius(maxlev)
                                      ! microwave and
      real effangmw
      real seceffmw
      real effangir
                                     ! IR channel zenith angles and secants
      real seceffir
                                      ļ
      real effang
      real seceff
      real h2ocd (maxlev)
                                     ! H2O column density
      real e(maxlev)
                                      ! H2O vapor pressure
      real dqh2ocd(maxlev)
                                      ! deriv. of column density wrt spec.humid.
      real dqe(maxlev)
                                      ! deriv. of vapor press wrt spec.humid.
! effective mean layer transmittance from fixed gas contribution
! and logarithmic derivative w.r.t. temperature
      real taufix(maxlev,maxangs)
```

```
real dttaufix(maxpack,maxangs)
! effective mean layer transmittance from water vapor contribution
! and logarithmic derivative w.r.t. temperature, specific humidity
     real tauh2o(maxlev,maxangs)
     real dqtauh2o(maxlev,maxangs), dttauh2o(maxlev,maxangs)
! effective mean layer transmittance from ozone contribution
! and logarithmic derivative w.r.t. temperature, ozone column density
     real tauozo(maxlev,maxangs)
     real dutauozo(maxlev, maxangs), dttauozo(maxlev, maxangs)
! effective mean layer transmittance at rapid algorithm angles and
! logarithmic derivatives w.r.t temperature, specific humidity,
! and ozone column density
     real tauang(maxlev,maxangs)
     real dttauang(maxpack,maxangs)
     real dqtauang(maxlev,maxangs)
     real dutauang(maxlev,maxangs)
! derivatives (w.r.t. temperature, specific humidity, and ozone) of
! effective mean layer transmittances along path from satellite zenith
! angle and sun angle
     real dttau(maxpack), dtsun(maxlev)
     real dqtau(maxlev), dqsun(maxlev)
     real dutau(maxlev), dusun(maxlev)
! calculated tau, tausun, derivatives and brightness temperature
! for current channel
     real ctau(maxlev), ctausun
     real cdTbdT(maxlev), cdTbdq(maxlev), cdTbdu(maxlev)
     real cbt
! flag for solar calculation
     logical dosun
      errflag = .false.
! calculate quantites which are common for all channels
! tav - mean layer temperature for RT calculation
! tdif - difference from rapid algorithm's std. atmos. mean temp.
! celsius - difference from 273 K
     (note this is not Celsius; 273 K is used in rapid algorithm)
     tav(1) = 0.5 * (tmptop + tair(1))
     tdif(1) = tav(1) - tstd_av(1)
      celsius(1) = tav(1) - 273.
     do l = 2, nlev
         tav(l) = (tair(l-1)+tair(l)) * 0.5
          tdif(1) = tav(1) - tstd_av(1)
          celsius(1) = tav(1) - 273.
      enddo
```

```
compute mid-level h2o column densities (g/cm**2)
   delprs = {\delta p } * 10 / grav --> density(air) * {\delta z}
   delprs * h2omid(spec. humid) --> column density effh2o U(1)
     do l = 1, nlev-1
       h2omid(1) = amax1(h2omid(1), shmin)
       h2ocd(1) = delprs(1) * h2omid(1)
       dqh2ocd(1) = delprs(1)
      enddo
     h2ocd(nlev) = (ps - pres(nlev-1)) * gravcon * h2omid(1)
     dqh2ocd(nlev) = (ps - pres(nlev-1)) * gravcon
! e(1) = H2O partial pressure (in atm)
           = (airmol/h2omol) * P(1) / (1013.25 mb per atm)
                       * h2omid (spec. humidity)
Ţ
ļ
     econ = airmol / (h2omol * atmosph) ! set in phycons.h
     do l = 1, nlev-1
        e(1) = presa(1) * econ * h2omid(1)
        dqe(1) = presa(1) * econ
      enddo
      e(nlev) = (ps + pres(nlev-1)) * 0.5 * econ * h2omid(nlev)
      dqe(nlev) = (ps + pres(nlev-1)) * 0.5 * econ
! set up zenith angle factors for microwave and IR channels
      effangmw = abs( deg2rad * mw_zen )
      seceffmw = 1. / cos( effangmw )
      effangir = abs( deg2rad * ir_zen )
      seceffir = 1. / cos( effangir )
! loop over requested channels
     do ichan = 1,nchan
        chan = chanarr(ichan)
        if (chan .gt. maxcha) then
           print *,' tovs_tau: bad channel requested'
           errflag = .true.
           return
         else
! set appropriate zenith angle (effang) and secant for IR or MW channel
            if (irchan(chan)) then
              effang = effangir
               seceff = seceffir
            else if (mwchan(chan)) then
               effang = effangmw
               seceff = seceffmw
```

endif

```
! set flag to show whether solar radiation calculations are performed
           dosun = ( freq(chan) .gt. 2000. ) .and. dayflag
! Calculate logarithmic derivatives of effective mean layer transmittances
! for fixed gas, water vapor and ozone contributions. The logarithmic
! derivatives are used because total transmittance from top of atmosphere
! to a level is the -product- of layer values of fixed gas, water vapor
! and ozone layer contributions.
! calculate contribution to effective layer transmittances from fixed gases
   (taufix) and logarithmic derivative of taufix w.r.t.
    layer temperature
                       1/taufix d(taufix)/dTavg : dttaufix
           call tovsfix_tauK(tdif, chan, nlev, taufix, dttaufix)
  calculate contribution to effective layer transmittances from water vapor
  (tauh2o) and logarithmic derivatives of tauh2o w.r.t.
      specific humidity
                         1/tauh2o d(tauh2o)/dq
                                                   : dqtauh2o
                                                                 and
     layer temperature
                          1/tauh2o d(tauh2o)/dTavg : dttauh2o
           call tovsh2o_tauK( h2ocd, e, dqh2ocd, dqe, chan, nlev,
               tav, celsius, tauh2o, dqtauh2o, dttauh2o)
! calculate contribution to effective layer transmittances from ozone
! (tauozo) and logarithmic derivative of tauozo w.r.t.
     ozone column density 1/tauozo d(tauozo)/d
į
           call tovsozo_tauK( ozomid, chan, nlev, celsius, tauozo,
                  dutauozo, dttauozo)
! take product of fixed, h2o, and ozone contributions to get
   mean layer transmittances & logarithmic derivatives for
   rapid algorithm angles
           call tovs_tauprodK(nlev, taufix, tauh2o, tauozo,
                dqtauh2o, dttauh2o, dutauozo, dttauozo, dttaufix,
    &
                tauang, dqtauang, dutauang, dttauang)
    &
! interpolate transmittance to observed zenith angle
 and calculate consistent logarithmic derivative terms
!
           call tovs_tauangK(tauang, chan, nlev, effang, seceff,
    &
              dosun, secsun, dqtauang, dutauang, dttauang,
    &r.
              ctau, ctausun, dutau, dusun, dqtau, dqsun,
              dttau, dtsun)
! calculate temperature, moisture and ozone Jacobians
   and brightness temperature
           call dTb_dTQU(ctau,ctausun,chan,tair,tmptop,ir_zen,
              emissmw, land, nlev, rho, secsun, dosun, tground,
    &
```

```
&
               tsurfair, dttau, dqtau, dutau, dtsun, dqsun, dusun,
               cbt, cdTbdT, cdTbdq, cdTbdu)
! fill arrays with calculated brightness temperature, tau and tausun
   and derivatives
             do l = 1, nlev
               tau(1,chan) = ctau(1)
                dTbdT(1,chan) = cdTbdT(1)
                dTbdq(1,chan) = cdTbdq(1)
                dTbdu(1,chan) = cdTbdu(1)
             enddo
             do l = nlev+1,maxlev
                tau(1,chan) = ctau(nlev)
                dTbdT(1,chan) = 0.0
               dTbdq(1,chan) = 0.0
                dTbdu(1,chan) = 0.0
             enddo
             tausun(chan) = ctausun
             bt(chan) = cbt
          endif
      enddo
                                          ! end of channel loop
     return
      end
```

tovs_tauangK.f - Interpolate transmittance to zenith angle

```
!!TOVS TAUANGK
!!ROUTINE: tovs_tauangK
!!DESCRIPTION: Interpolate computed transmittances to satellite
                zenith angle and combine transmittance derivatives
                consistent with angle interpolation and tau calculation
! CALLED FROM:
               tovs_tauK
! SYSTEM ROUTINES USED: exp, cos, acos
! SUBROUTINES CALLED: none
!!INPUT PARAMETERS:
! tauang - real matrix (of size maxlev by maxangs) of mean layer
              transmittance calculated for different zenith angles
! chan
          - integer channel number
! nlev
          - integer surface level (also number of levels to use)
! effang - real abs(zenith angle) in radians
         - real 1. / cos(effang)
! seceff
          - logical flag .true. if calculations for sun are to be done
 dosun
  secsun - real secant of solar angle for this observation
! dqtauang - real array (of size maxlev by maxangs) of derivative of
              tauang wrt. H2O specific humidity
 dutauang - real array (of size maxlev by maxangs) of derivative of
              tauang wrt. ozone
  dttauang - real array (of size maxpack by maxangs) of derivative of
              tauang wrt. mean layer temperature
                    first dimension: temperature layer
  packed matrix:
                                                       (see below)
                   second dimension: transmittance layer
!!OUTPUT PARAMETERS:
          - real array (of length maxlev) of mean layer transmittance
                for satellite zenith angle
  tausun - real mean layer transmittance
                for solar radiation
  dqtau
          - real array (of size maxlev) of logarithmic derivatives of
                 tau with respect to changes in moisture at given layers
                  -these apply to taus at all levels below the given layer
          - real array (of size maxlev) of logarithmic derivatives of
  dutau
                 tau with respect to changes in ozone at given layers
                  -these apply to taus at all levels below the given layer
  dttau
          - real array (of size maxpack) of logarithmic derivatives of
                 tau with respect to changes in temperature at given layers
                    first dimension: temperature layer
                                                       (see below)
  packed matrix:
                   second dimension: transmittance layer
          - real array (of size maxlev) of logarithmic derivatives of
              suntau with respect to changes in moisture at given layers
```

```
- real array (of size maxlev) of logarithmic derivatives of
! dusun
               suntau with respect to changes in ozone at given layers
  dtsun
           - real array (of size maxlev) of logarithmic derivatives of
               suntau with respect to changes in temperature at given layers
!!REVISION HISTORY:
  15dec95 Meta S. original routine derived from subroutine tauang
                    (derivs. based on earlier routine w/o interpolation)
  20dec95 Meta S. add changes to pass in temperature derivatives
                      consider only temperature deriv from B term
                      of tovsfix_tau (which would be 1/taufix *taucfb)
  08jan96 Meta S. include 'C' term derivative; dttaufix and dttau are
                     now dependent on tau level also
  12jan96 Meta S. move derivative products to tovstau_prod.f (for
                      clarity, mostly), edit prologue
  17jan96 Meta S. array notation in dttauang, dttau now consistent;
                      sum of temp. derivatives over transmittance layers
                      now back in this routine after interpolation (where
                      it should be!)
  16feb96 Meta S. minor optimization: swap dimensions of dttau*
  29mar96 Meta S. another minor optimization: change dttaufix
                    into packed array ((maxlev*(maxlev+1))/2,maxangs)
! Matrix Packing
                                (based on SPPTRF manual page)
! The j-th column of A is stored in the array AP as follows:
  AP(i + (j-1)*j/2) = A(i,j) \text{ for } 1 \le i \le j
     Two-dimensional storage of the matrix A:
į
       a11 a12 a13 a14
                         1st dimension: affecting temperature layer
        0 a22 a23 a24
                         2nd dimension: affected transmittance layer
        0 0 a33 a34
        0 0 0 a44 a12: influence of T(1) on \hat{t}(tau)(2)
        | zeros because temperatures in a given layer don't affect
             transmittance layers above that layer
    Packed storage of the upper triangle of A:
    AP = [a11, a12, a22, a13, a23, a33, a14, a24, a34, a44]
subroutine tovs_tauangK(tauang, chan, nlev, effang, seceff,
              dosun, secsun, dqtauang, dutauang, dttauang,
    &<del>.</del>
              tau, tausun, dutau, dusun, dqtau, dqsun,
              dttau, dtsun)
     implicit none
     include "tovsparam.h"
     integer maxpack
```

```
parameter (maxpack = (maxlev*(maxlev+1))/2 )
! variables passed into routine
     real tauang(maxlev,maxangs)
     integer chan
     integer nlev
     real
              effang
     real
              seceff
     logical dosun
     real
              secsun
     real
              dqtauang(maxlev,maxangs)
              dutauang(maxlev,maxangs)
     real
              dttauang(maxpack,maxangs)
     real
! variables output from routine
     real
              tau(maxlev)
     real
              tausun
     real
              dutau(maxlev)
              dusun(maxlev)
     real
     real
              dqtau(maxlev)
     real
              dqsun(maxlev)
            dttau(maxpack)
     real
             dtsun(maxlev)
     real
! constants, quasi-constants, and coefficients
! (in common block or passed into routine)
     include "taucoef.h"
     include "grdcons.h"
     include "chacons.h"
! local variables
     real
            anglsun
     real
             efcos
     real
              efsun
     integer k, l, m, n, nm1, nnm1
     integer nangm1
     integer nfang
     integer nfsun
     real
            sat_factor
             sfactor
     real
     real
              sun_factor
     real
              tauscat
              tauint
                                 ! interpolated tau: \tilde{tau}(\theta)
     real
                                ! 1. / \tilde{tau}(\theta)
     real
              tauinti
! statement functions
     real
              quikexp
     real
     quikexp(X) = 1.0 + X * (1.0 + X * (.5 + X * .16666667))
     nangm1 = numangs - 1
     tausun = 0.
                       ! set this in case of .not.irchan or .not.dosun
     do l = 1, maxlev
```

```
dusun(1) = 0.
       dqsun(1) = 0.
       dtsun(1) = 0.
      enddo
! find which angles of rapid algorithm bracket the observed zenith angle
  and calculate factor for interpolation in sec(zenith angle)
  first consider transmittance layer 1, then loop over rest of layers
     nfang = 1
     do k = 2, nangm1
         if (effang .gt. angle(k) ) nfang = k
      enddo
     sat_factor = ( seceff - secang(nfang) ) /
                       (secang(nfang+1) - secang(nfang) )
     tau(1) = amax1(0.0, (tauang(1, nfang))
          + sat_factor * (tauang(1,nfang+1)-tauang(1,nfang)) ) )
      if (tau(1).eq. 0.0) then
         dqtau(1) = 0.0
        dutau(1) = 0.0
         dttau(1) = 0.0
      else
        tauinti = 1. / tau(1)
        dqtau(1) = tauinti *
              ((1. - sat_factor)*dqtauang(1,nfang)
    &
    &r.
                   + sat_factor*dqtauang(1,nfang+1))
        dutau(1) = tauinti *
              ((1. - sat_factor)*dutauang(1,nfang)
    &
                   + sat_factor*dutauang(1,nfang+1))
     &
        dttau(1) = tauinti *
    &
                ((1. - sat_factor)*dttauang(1,nfang)
     &
                      + sat_factor *dttauang(1,nfang+1))
     endif
! loop over transmittance layers
     do l = 2, nlev
        n = (1-1)*1/2
                               ! set index for packed array
        nm1 = (1-2)*(1-1)/2 ! for levels 1 and 1-1
         nnm1 = n - nm1
        tauint = amax1( 0.0, (tauang(l, nfang)
               + sat_factor * (tauang(l,nfang+1)-tauang(l,nfang))))
         tau(1) = tau(1-1) * tauint
  calculate factors dqtau ( 1/tau d(tau)/dq) and dutau (1/tau d(tau)/du)
```

```
if (tauint .eq. 0.0 ) then
            dqtau(1) = 0.0
            dutau(1) = 0.0
            do m = 1,1
               dttau(m+n) = 0.0
            enddo
         else
            tauinti = 1. / tauint
            dqtau(l) = tauinti *
    &
              ((1. - sat_factor)*dqtauang(l,nfang)
    &
                   + sat_factor *dqtauang(1,nfang+1))
            dutau(l) = tauinti *
              ((1. - sat_factor)*dutauang(l,nfang)
    &
                   + sat_factor *dutauang(1,nfang+1))
    &r.
            do m = 1,1-1
               dttau(m+n) = dttau(m+nm1) + tauinti *
                 ((1. - sat_factor)*dttauang(m+n,nfang)
    &
    &
                      + sat_factor *dttauang(m+n,nfang+1))
            do m = n+1, n+l-1
               dttau(m) = dttau(m-nnm1) + tauinti *
     &
                 ((1. - sat_factor)*dttauang(m,nfang)
                      + sat_factor *dttauang(m,nfang+1))
     &
            enddo
            dttau(l+n) = tauinti *
    &
               ((1. - sat_factor)*dttauang(l+n,nfang)
                    + sat_factor *dttauang(l+n,nfang+1))
     &
         endif
      enddo
      if (irchan(chan)) then
         dol=1,nlev
            sfactor = chscatt(chan) * scat(1)
            tauscat = quikexp( -sfactor * seceff )
            tau(1) = tau(1) * tauscat
         enddo
         if (dosun) then
! find which angles of rapid algorithm bracket the effective sun angle
! and calculate factor for interpolation in sec(sun angle)
            anglsun = seceff + secsun
            efcos = 1. / anglsun
            efsun = acos( efcos )
            nfsun = 1
            do k = 2, nangm1
               if ( efsun .gt. angle(k) ) nfsun = k
            enddo
```

```
sun_factor = ( anglsun - secang(nfsun) ) /
                          (secang(nfsun+1) - secang(nfsun) )
     &
! calculate product of transmittance in layers along solar path
            tausun = amax1( 0.0, (tauang(1,nfsun)
              + sun_factor * (tauang(1,nfsun+1) - tauang(1,nfsun)) ))
     &
            if (tausun .eq. 0.0) then
               dqsun(1) = 0.0
               dusun(1) = 0.0
               dtsun(1) = 0.0
            else
               tauinti = 1. / tausun
               dqsun(1) = tauinti *
                ((1.-sun_factor)*dqtauang(1,nfang)
     &
     &
                    +sun_factor *dqtauang(1,nfang+1))
               dusun(1) = tauinti *
                ((1.-sun_factor)*dutauang(1,nfang)
     &
     &
                    +sun_factor*dutauang(1,nfang+1))
               dtsun(1) = tauinti *
                ((1.-sun_factor)*dttauang(1,nfang)
     &
                    + sun_factor*dttauang(1,nfang+1))
     &r.
            endif
            do 1 = 2, nlev
               n = (1-1)*1/2
                                      ! set index for packed array
               tauint = amax1( 0.0, (tauang(l, nfsun)
                 + sun_factor * (tauang(l,nfsun+1)-tauang(l,nfsun))))
     &
               tausun = tausun * tauint
               if (tauint .eq. 0.0) then
                  dqsun(1) = 0.0
                  dusun(1) = 0.0
                  dtsun(1) = 0.0
               else
                  tauinti = 1. / tauint
                  dqsun(1) = tauinti *
                   ((1. - sun_factor)*dqtauang(l,nfang)
     &
     &r.
                        + sun_factor *dqtauang(1,nfang+1))
                  dusun(1) = tauinti *
                   ((1. - sun_factor)*dutauang(l,nfang)
     &
     &
                        + sun_factor *dutauang(1,nfang+1))
                  do m = 1, 1-1
                     dtsun(m) = dtsun(m) + tauinti *
                       ((1. - sat_factor)*dttauang(m+n,nfang)
     &
                            + sat_factor *dttauang(m+n,nfang+1))
     Хr.
                  enddo
                  dtsun(1) = tauinti *
                     ((1. - sat_factor)*dttauang(l+n,nfang)
     &
```

```
# sat_factor *dttauang(l+n,nfang+1))

endif
enddo
sfactor = chscatt(chan) * scat(nlev)
tauscat = quikexp( -sfactor * anglsun )
tausun = tausun * tauscat !!! check this...
endif

endif
return
end
```

```
!!TOVS TAUPRODK
!!ROUTINE: tovs_tauprodK
!!DESCRIPTION: Calculate product of fixed, h2o and ozone contributions
                to transmittances and derivatives of tau product
                    ** with minor optimization
! CALLED FROM: tovs_tauK
! SYSTEM ROUTINES USED: none
! SUBROUTINES CALLED: none
!!INPUT PARAMETERS:
          - integer surface level (also number of levels to use)
  taufix - real array (of size maxlev by maxangs) of effective
                 mean layer transmittance for fixed gases
! tauh2o - real array (of size maxlev by maxangs) of effective
                 mean layer transmittance for moisture
! tauozo - real array (of size maxlev by maxangs) of effective
                 mean layer transmittance for ozone
! dqtauh2o - real array (of size maxlev by maxangs) of derivative
                 of tauh2o with respect to H2O specific humidity
  dttauh2o - real array (of size maxlev by maxangs) of derivative
                 of tauh2o with respect to temperature
  dutauozo - real array (of size maxlev by maxangs) of derivative
                 of tauozo with respect to ozone
! dttauozo - real array (of size maxlev by maxangs) of derivative
                 of tauozo with respect to temperature
! dttaufix - real array (of size maxpack by maxangs) of
                 derivative of taufix with respect to temperature
! packed matrix with first dimension: temperature layer
                  second dimension: transmittance layer
!!OUTPUT PARAMETERS:
          - real matrix (of size maxlev by maxangs) of mean layer
              transmittance calculated for different zenith angles
  dqtauang - real array (of size maxlev by maxangs) of derivative of
              tauang wrt. H2O specific humidity
 dutauang - real array (of size maxlev by maxangs) of derivative of
              tauang wrt. ozone
  dttauang - real array (of size maxpack by maxangs) of
              derivative of tauang wrt. mean layer temperature
                   first dimension: temperature layer
                                                       (see below)
 packed matrix:
                   second dimension: transmittance layer
!!REVISION HISTORY:
! 12jan96 Meta S. original routine
```

```
17jan96 Meta S. sum of temperature deriv over layers moved to
                    tovs_tauangK, *after* interpolation to zenith angle
                    removed TAUCFB, TAUCFC - added dttaufix input
                   minor optimization: Switched dimensions of dttaufix,
  16feb96 Meta S.
                       dttauang
                   another minor optimization: change dttaufix, dttauang
  29mar96 Meta S.
                    into packed array ((maxlev*(maxlev+1))/2,maxangs)
! Matrix Packing
                                (based on SPPTRF manual page)
  The j-th column of A is stored in the array AP as follows:
  AP(i + (j-1)*j/2) = A(i,j) \text{ for } 1 \le i \le j
į
     Two-dimensional storage of the matrix A:
       a11 a12 a13 a14
                         1st dimension: affecting temperature layer
        0 a22 a23 a24
                         2nd dimension: affected transmittance layer
        0
          0 a33 a34
          0 0 a44
                         a12: influence of T(1) on \hat(\tau)(2)
        | zeros because temperatures in a given layer don't affect
             transmittance layers above that layer
    Packed storage of the upper triangle of A:
    AP = [a11, a12, a22, a13, a23, a33, a14, a24, a34, a44]
subroutine tovs_tauprodK( nlev, taufix, tauh2o, tauozo,
                dgtauh2o, dttauh2o, dutauozo, dttaufix,
                tauang, dqtauang, dutauang, dttauang)
    &r.
     implicit none
     include "tovsparam.h"
     integer maxpack
     parameter (maxpack = (maxlev*(maxlev+1))/2 )
! variables passed into routine
     integer nlev
     real taufix(maxlev,maxangs)
     real tauh2o(maxlev,maxangs)
     real tauozo(maxlev, maxangs)
     real dqtauh2o(maxlev,maxangs)
     real dttauh2o(maxlev,maxangs)
     real dutauozo(maxlev,maxangs)
     real dttauozo(maxlev,maxangs)
     real dttaufix(maxpack,maxangs)
! variables passed out of routine
     real tauang(maxlev,maxangs)
     real dqtauang(maxlev,maxangs)
     real dutauang(maxlev,maxangs)
     real dttauang(maxpack, maxangs)
```

```
! local variables
      integer k,l,m,n
      do k = 1, maxangs
         do l = 1, nlev
           n = (1-1)*1/2
                              ! set index for packed array
            tauang(1,k) = taufix(1,k)*tauh2o(1,k)*tauozo(1,k)
            dqtauang(1,k) = tauang(1,k) * dqtauh2o(1,k)
            dutauang(1,k) = tauang(1,k) * dutauozo(1,k)
           do m = 1, 1
               dttauang(m+n,k) = tauang(l,k) * dttaufix(m+n,k)
           do m = n+1, n+1
               dttauang(m,k) = tauang(l,k) * dttaufix(m,k)
            enddo
            dttauang(l+n,k) = dttauang(l+n,k) +
    &
                      tauang(1,k) * (dttauh2o(1,k) + dttauozo(1,k))
         enddo
         do l = nlev+1,maxlev
           n = (1-1)*1/2
                              ! set index for packed array
           tauang(1,k) = 0.
           dqtauang(1,k) = 0.
            dutauang(1,k) = 0.
            do m = n+1, n+1
               dttauang(m,k) = 0.
            enddo
         enddo
      enddo
      return
      end
```